From Micro to Macro: The Influence of Firm Heterogeneity on Foreign Shock Transmission *

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February 2, 2024

Abstract

We investigate the role of firm heterogeneity and adjustment costs in the transmission of foreign supply shocks. Our starting point comes from a theoretical insight: If larger firms rely more on easily adjustable inputs, such as materials, then the aggregate output response to changes in the price of these inputs gets amplified relative to a representative firm economy. We next provide empirical evidence that larger firms are indeed more materials-intensive and more responsive to an exogenous foreign shock. We show that a New-Keynesian general equilibrium model with multiple sectors and firm heterogeneity is consistent with these facts. We find that firm heterogeneity, in line with the data, amplifies the response of output and prices to a foreign supply shock, but dampens the labor and GDP responses.

*We are indebted to our supervisors Jeppe Druedahl, Jakob R. Munch, and Søren Hove Ravn for their continuous guidance and support. We are grateful to Saroj Bhattarai, Christoph Boehm, Magnus T. Buus, Olivier Coibion, Francois de Soyres, Aaron Flaaen, Christian P. Hoeck, Andrei Levchenko, Isabelle Mejean, Carter Mix, Mads A. Nielsen, Nitya Pandalai-Nayar, Emil H. Partsch, Margit Reischer, Elisa Rubbo, Peter P. Stephensen, and Francesco Zanetti for valuable comments. We also thank seminar participants at UT Austin, the University of Copenhagen, Georgetown University, the Federal Reserve Bank Board, and the 2023 Annual SNEE Conference for questions and comments. Financial support from the Carlsberg Foundation (Grant CF20-0546) and the Innovation Fund Denmark (Grant 0196-00009B) is gratefully acknowledged. The opinions expressed are those of the authors alone.

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1 Introduction

Supply chain disruptions have become increasingly common in today's global economy. In this context, understanding how supply shocks are transmitted through supply chains is essential for policymakers and firms alike. Recently, it has been highlighted that large firms account for the majority of aggregate trade flows (Freund and Pierola, 2015; Di Giovanni et al., 2017; 2018; 2024). Thus, firm heterogeneity is key to understanding how shocks are transmitted across countries via value chains.

In this paper, we investigate how firm heterogeneity affects the transmission of foreign shocks when adjustment of production inputs is costly. We develop a New-Keynesian multi-sector heterogeneous firm model with labor adjustment costs, calibrated to the universe of Danish firms, featuring heterogeneity along several dimensions. We provide a novel theoretical insight: if larger firms rely more on easily adjustable inputs, such as materials, then the aggregate output and price response get amplified relative to a representative firm economy. Using the heterogeneous firm model, our main contributions are to show that i) firm heterogeneity amplifies the output and producer price responses to foreign supply shocks compared to a model with a representative firm within each sector. ii) Firm heterogeneity dampens the labor and GDP response to foreign shocks because large firms are less labor-intensive. Thus, our findings highlight that firm heterogeneity combined with adjustment costs is vital to properly understanding the transmission of foreign supply shocks.

Our starting point is a stylized heterogeneous firm model where firms differ in their size and their import and material shares, and where the adjustment of labor is costly. In this setting we provide a theoretical insight: Firm heterogeneity in size and materials shares amplify the output response to foreign shocks for two reasons. First, heterogeneity in the exposure to shocks (variance of material shares) generates an amplification of the shock. Second, when the largest firms rely on flexible production inputs, i.e., a positive correlation between output and material shares, the shock get further amplified. The conditions for this result is a positive labor adjustment cost. Thus, as in Baqaee and Farhi (2019b), non-linearities in the microeconomic production structure are required to generate amplification. We also show that for a reasonable calibration, the labor response is dampened by firm heterogeneity, no matter the presence of adjustment costs. This last prediction is consistent with Di Giovanni et al. (2024) finding a dampening response in GDP in a model absent any rigidities.

We next provide empirical validation of these theoretical predictions by presenting three stylized facts about the universe of Danish firms. Within sectors, larger firms are systematically more materials-, import-, and export-intensive. Thus, a model with only sectoral heterogeneity misses out on a potentially important source of shock amplification. Second, we apply a shift-share design to study how firms of different sizes respond to an exogenous foreign supply shock. Larger firms are considerably more affected by foreign shocks. Following a supply shock corresponding to a 10% increase in import prices, firms at the first (third) quartile of the size distribution reduce their sales by 3% (10%). Lastly, the materials response to a foreign supply shock is more than double as large as the labor response and less persistent. This suggests that labor is costly to adjust.

Motivated by these facts, we present a general equilibrium model calibrated to the universe of Danish firms. The model is a standard New Keynesian small open economy model, augmented with firm and sector heterogeneity and adjustment costs. Firms are heterogeneous along four dimensions: i) Material share in total expenditures, ii) import shares, iii) export shares, and iv) size. Consistent with the stylized facts, larger firms are more materials-, import-, and export-intensive. Importantly, the model closely replicates our second stylized fact, i.e., the heterogeneity in output responses across the firm size distribution.

We begin our analysis by considering a stylized supply shock corresponding to a 10% increase in import prices. In partial equilibrium, the heterogeneous firm model predicts an 8% drop in output on impact and an increase in prices of around 1% consistent with our empirical evidence. Compared to a model with only sector heterogeneity, we find that adding firm heterogeneity amplifies the output response to the foreign shock considerably. This finding owes to the fact that larger firms are also more materials-intensive and therefore rely on more flexible production inputs. At the same time, heterogeneity in material shares implies that firms are differently exposed to shocks, which creates a further amplification. The fact that larger firms are also more materials-intensive implies that labor-intensive firms are mainly small firms. Therefore, firm heterogeneity dampens the labor response to foreign shocks as the firms most exposed to foreign shocks are less connected to the domestic labor market.

These conclusions carry, by and large, over to general equilibrium, where a multitude of new channels exists such as changes in the wage, demand, and competitor prices. In particular, we find that firm heterogeneity amplifies the inflation response by around 100% in general equilibrium. Because the larger firms are more materials-intensive, they are more likely to pass on the shock downstream to other domestic firms, which increases the inflation response further. Because the larger firms are less connected to the labor market, firm heterogeneity dampens the response in GDP by around 20%. The dampening in the GDP response is primarily driven by the impact of the import price, i.e., the same transmission channel as in partial equilibrium.

Taken together, our results show that firm heterogeneity in itself is not sufficient to generate an amplification in the output and price response. However, when combined with non-linearities in the form of adjustment costs, output and prices get amplified considerably, with prices increasing double the amount in a model with a representative firm. In light of recent events in Europe with sharp increases in prices, these effects seem quantitatively important for firms and policymakers trying to predict inflation.

Contributions and related literature. Our first and main contribution is to illustrate the role of firm heterogeneity in shock transmission when adjustment costs are present. We illustrate the dampening effect on GDP from firm heterogeneity, providing support for the results in Di Giovanni et al. (2024). The authors set up a static multi-sector model for France with heterogeneous firms. We add to their paper in several aspects. The first and most important deviation is the presence of adjustment costs. We show that with adjustment costs, the inflation and output response to foreign shocks get amplified. Transmission of inflationary shocks is a topic that has recently gained extra attention in the aftermath of the global energy crisis (Amiti et al., 2023; Raphaël Lafrogne-Joussier et al., 2023; Ferrante et al., 2023). Thus, we extend the analysis beyond GDP dynamics. Second, our model is calibrated to match empirical evidence on the firm-level influence of a foreign supply shock and matches the heterogeneity in output response along the firm-size distribution. Third, our model is fundamentally different in that it is a dynamic New Keynesian model with nominal and real rigidities. As highlighted by Ho et al. (2022), nominal rigidities are key for analyzing the transmission of shocks across countries.

Second, we contribute to the literature on shock propagation through production networks (Long Jr and Plosser, 1983; Carvalho, 2008; Acemoglu et al., 2012; Johnson, 2014; Barrot and Sauvagnat, 2016; Baqaee and Farhi, 2019a; Huo et al., 2023; Reischer et al., 2019; Carvalho and Tahbaz-Salehi, 2019; La'O and Tahbaz-Salehi, 2022; Foerster et al., 2022; Vom Lehn and Winberry, 2022; Rubbo, 2023; Afrouzi and Bhattarai, 2023). Most related to our paper is Baqaee and Farhi (2019b) who illustrate the role of microeconomic production structures in amplifying shocks. We build on these insights by including adjustment costs and illustrate the interplay between firm size and exposure to foreign shocks for shock amplification. Also, we highlight how firm heterogeneity in import (materials) shares dampens (amplifies) the transmission of shocks through the domestic production network.

Our last contribution is identifying the dynamic effects of several firm-level variables to an exogenous foreign supply shock. In that sense, we relate to a set of papers investigating the impact of supply shocks using an exogenous variation, such as shift-share instruments (Huneeus, 2018; Huneeus et al., 2021) or natural disasters like the COVID-19 pandemic (Meier and Pinto, 2020; Raphael Lafrogne-Joussier et al., 2022) or the 2011 Thoku earthquake in Japan (Boehm et al., 2019; Carvalho et al., 2021). We add to this literature by estimating the dynamic impact on several firm-level variables of a supply shock. Importantly, we provide novel evidence of the heterogeneity in dynamic response to supply shocks across the firm-size distribution. In addition, Our empirical analysis of foreign cost-push shocks and the pass-through of marginal costs to prices add to an abundant literature (Gopinath and Itskhoki, 2010; Nakamura and Zerom, 2010; Fabra and Reguant, 2014; Amiti et al., 2019; Phelan and L'Huillier, 2023).

Additionally, our paper focuses on firm heterogeneity. Starting with Melitz (2003), there is a large literature focusing on the interaction between trade and heterogeneity with a specific focus on sorting based on productivity (Greenaway and Kneller, 2007; Melitz and Ottaviano, 2008; Baldwin and Harrigan, 2011; Johnson, 2012; Kugler and Verhoogen, 2012; Antoniades, 2015; Feenstra, 2018). Similarly, there is a big literature in macroeconomics that highlight firm heterogeneity in the propagation of aggregate shocks, typically with an emphasis on price rigidities, financial frictions or uncertainty (Bloom, 2009; Nakamura and Steinsson, 2010; Khan and Thomas, 2013; Buera and Moll, 2015; Huneeus, 2018; Arellano et al., 2019; Ottonello and Winberry, 2020). Closely related to this literature, a newer literature - starting with the the seminal contribution by Gabaix (2011) - highlights the importance of large firms in the propagation of aggregate shocks (see also Acemoglu et al., 2012; Freund and Pierola, 2015; Di Giovanni et al., 2017; 2018).

2 Firm Heterogeneity, Rigidity, and Shock Amplification

Our starting point is to illustrate the interplay between firm heterogeneity and adjustment costs in generating amplification in output but dampening of labor. In Section 2.1, we consider a stylized heterogeneous firm model with adjustment costs in labor. Firms differ in their size, their materials share, and their import share. Section 2.2 illustrates the amplifying effect of firm heterogeneity when adjustment costs are present. In Section 2.3, we show that the effect of firm heterogeneity on the labor response to foreign shocks is ambiguous due to opposing forces.

2.1 A Stylized Heterogeneous Firm Model

Production. Firm i produce output, z_i, using CES technology over total labor $\hat{\ell}_i$ and materials m_i:

$$z_i = \left[\alpha_i^{\frac{1}{\phi}} m_i^{\frac{\phi-1}{\phi}} + (1-\alpha_i)^{\frac{1}{\phi}} \hat{\ell}_i^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{1}$$

where $0 \leq \alpha_i \leq 1$ is a firm-specific material expenditure share and $\phi > 0$ is the elasticity of substitution between materials and labor. We follow Baqaee and Farhi (2019b) in modeling a reduced form specification that captures the limited ability of firms to adjust their labor input in the short run. Total labor is a geometric average of flexible labor input ℓ_i and fixed labor $\overline{\ell}_i$:

$$\hat{\ell}_i = \ell_i^{1-\omega} \overline{\ell}_i^{\omega} \tag{2}$$

Here ω measures the share of labor that is immediately adjustable in response to shocks. With $\omega = 1$ the input of labor is fully inelastic at the steady state level $\overline{\ell}_i$, whereas $\omega = 0$ implies that firms can adjust the entirety of the labor input. In practice, this implies that in the short run, we have a lower return to scale to labor whenever $\omega > 0$.

Labor is rented from households at the going wage rate W, taken as given by individual firms. The input of materials, m_i, is a CES aggregate of imported and domestic materials:

$$m_i = \left[\gamma_i^{\frac{1}{\vartheta}} \left(m_i^F \right)^{\frac{\vartheta-1}{\vartheta}} + (1-\gamma_i)^{\frac{1}{\vartheta}} \left(m_i^D \right)^{\frac{\vartheta-1}{\vartheta}} \right]^{\frac{\vartheta}{\vartheta-1}} \tag{3}$$

where $0 \leq \gamma_i \leq 1$ is a firm-specific import share. The firm-specific material price is:

$$P_i^M = \left[\gamma_i \left(P^{M,F} \right)^{1-\vartheta} + (1-\gamma_i) \left(P^{M,D} \right)^{1-\vartheta} \right]^{\frac{1}{1-\vartheta}}$$

Note that all firms face the same prices of foreign and domestic materials $P^{M,F}, P^{M,D}$, but the overall material price P_i^M is firm-specific because of heterogeneity in import shares.

Sales and price setting. Firms sell their products in a common market, competing against all other firms. Firms face the following CES demand function:

$$z_i = \varrho_i \left(\frac{p_i}{P} \right)^{-\epsilon^P} Z \tag{4}$$

where Z is aggregate demand, P is the aggregate price index, p_i is the price set by firm i, ϱ_i is a firm-specific demand-shifter and $\epsilon^P > 1$ is the elasticity of substitution between firm-level products. We assume that inputs are subsidized at rate τ as in Galí (2015), and set $\tau = \frac{1}{\epsilon^P}$ to eliminate the distortion arising from markups.[1] Profits are given by:

$$\Pi_i = p_i z_i - (1-\tau) W \ell_i - (1-\tau) P_i^M m_i \tag{5}$$

1. We do this only to facilitate the analytical exposition below, as this allows us to focus on an initial equilibrium in which all prices are equalized.

The problem of the firm is to pick $\{p_i, z_i, \ell_i, m_i, m_i^D, m_i^F\}$ so as to maximize (5) subject to constraints (1)-(4).

Aggregation. Total production is given by the CES aggregate:

$$Z = \left[\int (\varrho_i)^{\frac{1}{\epsilon^P}} (z_i)^{\frac{\epsilon^P - 1}{\epsilon^P}} \, \mathrm{d}i \right]^{\frac{\epsilon^P}{\epsilon^P - 1}}$$

with the associated price index:

$$P = \left[\int \varrho_i (p_i)^{1-\epsilon^P} \, \mathrm{d}i \right]^{\frac{1}{1-\epsilon^P}}$$

2.2 Amplification of Output Response

To more clearly understand the role of heterogeneity in the transmission of a foreign supply shock, we here derive some analytical insights. We consider the change in firm sales based on a negative foreign supply shock, measured as an increase in the import price $dP^{M,F}$. The linearized response of firm-level output to an increase in the import price is given in lemma 1:

Lemma 1. *The response of output for firm i with steady state characteristics $\{\alpha_i, \gamma_i, z_i\}$ to a change in the import price $P^{M,F}$ is given by:*

$$dz_i = -\epsilon^P \phi \frac{\alpha_i \gamma_i z_i}{\omega \epsilon^P + \phi (1-\omega) + \alpha_i \omega (\phi - \epsilon^P)} dP^{M,F} \tag{6}$$

and the aggregate response of output is:

$$dZ = -\epsilon^P \phi \int \frac{\alpha_i \gamma_i z_i}{\omega \epsilon^P + \phi (1-\omega) + \alpha_i \omega (\phi - \epsilon^P)} \, \mathrm{d}i \, dP^{M,F} \tag{7}$$

Proof: appendix XX.

The lemma establishes that firms reduce output in response to higher import prices and that the size of the response scales positively with shock exposure as measured by the steady level of total imports, given by $\alpha_i \gamma_i z_i$. The lemma characterizes the response for a general level labor adjustment, and with an arbitrary level of heterogeneity in $\{\alpha_i, \gamma_i, z_i\}$. Proposition 1 establishes the relationship between a heterogeneous firm model with general $\{\alpha_i, \gamma_i, z_i\}$ and the standard representative firm model in the case of perfect labor adjustment:

Proposition 1 (Equivalence)**.** *If labor is perfectly adjustable $\omega = 0$ or $\phi = \epsilon^P$ the response of firm-level and aggregate output is simply:*

$$dz_i = -\epsilon^P \alpha_i \gamma_i z_i dP^{M,F}$$
$$dZ = -\epsilon^P \int \alpha_i \gamma_i z_i \,\mathrm{d}i\, dP^{M,F}$$

The response in the representative firm model is:

$$dZ^{RA} = -\alpha^{RA} \gamma^{RA} \cdot Z \cdot dP^{M,F}$$

If we focus on the representative firm model that produces the same steady state macro levels $\alpha^{RA} = \frac{\int m_i di}{\int z_i di}$, $\gamma^{RA} = \frac{\int m_i^F di}{\int m_i di}$, the micro level heterogeneity is irrelevant for the aggregate response:

$$dZ^{RA} = dZ$$

The proposition establishes that in the frictionless benchmark, heterogeneity is irrelevant for the response of aggregate output, and the heterogeneous and representative firm model coincide. Figure 1 displays this for a stylized calibration. At $\omega = 0$, the output responses are equalized across the models, but moving away from the benchmark this is generally not the case.

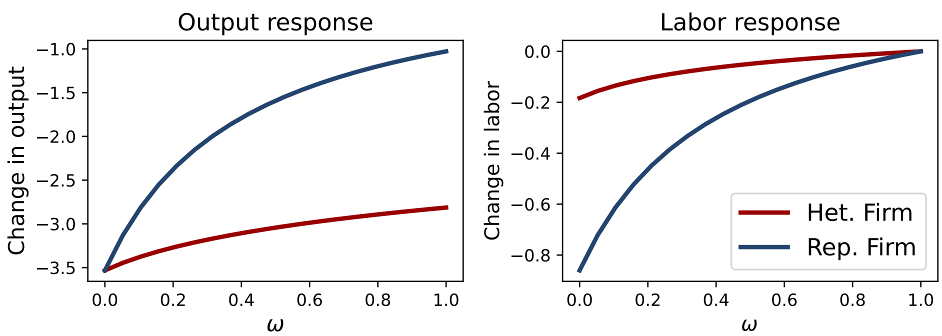


Figure 1: Stylized Example: Change in Output and Labor as a Function of Labor Adjustment Costs

Note: The figure displays the change in output dZ and labor dL as a function of the degree of labor adjustment ω for the representative and heterogeneous firm models. The other parameters are $\phi = 1, \epsilon^P = 9, dP^{M,F} = 1, \alpha^{RA} = 0.7, \gamma^{RA} = 0.5$.

Having established under what circumstances heterogeneity matters for the response of output, we next ask if heterogeneity amplifies or dampens the aggregate response. Proposition 2 answers this question:

Proposition 2 (Heterogeneity and the response of aggregate output.)**.** *Assume $\gamma_i = \overline{\gamma}\ \forall\ i$, and let $\overline{x} = \mathbb{E}_i[x]$ for some x. Then, utilizing a second-order approximation of the* *coefficient in (7), the response of aggregate output is:*

$$dZ = -\frac{\epsilon^P \phi}{\overline{\psi}} \overline{\alpha \gamma} Z dP^{M,F} - \frac{\epsilon^P \phi \overline{\gamma z} \left(\omega \epsilon^P + \phi \left(1 - \omega\right)\right) \omega \left(\epsilon^P - \phi\right)}{\overline{\psi}^4} \mathbb{V}\text{ar}\left(\alpha_i\right) dP^{M,F}$$
$$- \frac{\epsilon^P \phi \overline{\gamma} \left(\omega \epsilon^P + \phi \left(1 - \omega\right)\right)}{\overline{\psi}^2} \mathbb{C}\text{ov}\left(\alpha_i, z_i\right) dP^{M,F}$$

where $\overline{\psi} > 0$ is a function of steady state parameters, see appendix B.3.

The proposition shows that amplification in the output response is driven by two sources of heterogeneity, the first being the variance of material shares and the second the covariance between output and material shares. However, this amplification depends on the presence of frictions in the reallocation of factor inputs and $\phi \neq \epsilon^P$. This result is related to the Hulten (1978)'s theorem. The theorem states that to a first-order approximation, aggregate TFP growth is given by the microeconomic TFP growths weighted by their sales shares (i.e., the Domar weights). This result has led macroeconomists to de-emphasize the role of microeconomic heterogeneity. The condition for this result is that production is Cobb-Douglas, as this keeps the Domar weights constant.

Importantly, Baqaee and Farhi (2019b) show that with non-linearities in the production structure, such as a non-unitary production structure, decreasing returns to scale, or labor adjustment costs, the theorem breaks down and firm heterogeneity matters for the aggregate response. Consistent with this result, proposition 2 shows that when the economy deviates from Cobb Douglas, i.e., $\phi \neq 1$ and $\epsilon^P \neq 1$ (or more generally $\phi \neq \epsilon^P$) and adjusting labor is costly ($\omega > 0$), firm heterogeneity creates an amplification of the output response.

In proposition 2, we assumed no heterogeneity in import intensities for simplicity. As shown in corollary 1, heterogeneity in import intensity does not affect the response of output in and of itself.

Corollary 1 (Irrelevance of firm size and import intensiveness)**.** *Consider a model with heterogeneity in firm size and import intensity ($\mathbb{V}\text{ar}\left(\gamma_i\right) > 0, \mathbb{V}\text{ar}\left(z_i\right) > 0$) but a common material intensity in all firms $\alpha_i = \alpha \; \forall i$ ($\mathbb{V}\text{ar}\left(\alpha_i\right) = 0$). Then:*

$$dZ^{RA} = dZ$$

Even if labor is not perfectly adjustable $\omega > 0$.

Notice that because of demand curve (4) all the above results for output also carry over to the response of prices dp_i in our model when considering micro-level shocks which keeps aggregate demand and prices fixed.

2.3 Dampening of Labor Response

Though proposition 1 seems to imply equivalence between the representative firm model and more general models when $\omega = 0$, this does not apply to labor and material demand. As shown in proposition 3, the response of labor in the heterogeneous firm model to a foreign import shock equals the rep. firm response dL^{RA} plus a term capturing micro-level variation in the use of labor and materials. This term will be negative for firms that rely less on materials, but positive for firms that use a lot of materials. Whether the term is positive or negative in the aggregate depends on the joint distribution of $\{\alpha_i, \gamma_i, z_i\}$. In the case where larger firms use more materials, the term will tend to be positive, implying a lower response of labor in the heterogeneous firm model compared to the rep. firm model.

Proposition 3 (Labor responses). *The response of firm level labor is:*

$$d\ell_i = -\phi \frac{(1-\omega)(1-\alpha_i)\left(\epsilon^P - \phi\right)}{\epsilon^P \omega + \phi(1-\omega) - \alpha_i \omega\left(\epsilon^P - \phi\right)} \gamma_i \alpha_i z_i dP^{M,F} \tag{8}$$

Assume $\omega = 0$. *Then the response of aggregate labor is:*

$$dL = dL^{RA} + \left(\epsilon^P - \phi\right) \int \alpha_i z_i \gamma_i \left\{\alpha_i - \alpha^{RA}\right\} \mathrm{d}i \, dP^{M,F}$$

where:

$$dL^{RA} = -\left(\epsilon^P - \phi\right)\left(1 - \alpha^{RA}\right) \alpha^{RA} \gamma^{RA} Z dP^{M,F}$$

We next diverge from the case of no adjustment cost, and instead consider the case with positive adjustment costs, $\omega > 0$, and derive a general expression for the response of labor in proposition 4.

Proposition 4 (Firm heterogeneity and the response of labor.). *Utilizing a second-order approximation of the coefficients in* (8), *the response of aggregate labor is:*

$$\begin{aligned}
dL \approx & -\phi \frac{(1-\omega)\left(\epsilon^P - \phi\right)(1-\overline{\alpha})\,\overline{\alpha\gamma z}}{\overline{\psi}} dP^{M,F} \\
& -\phi(1-\omega)\left(\epsilon^P - \phi\right) \frac{\overline{\gamma z}}{\overline{\psi}^2}\left(\omega\epsilon^P + \phi(1-\omega)\right)\left[\frac{(1-\overline{\alpha})\,\omega\left(\epsilon^P - \phi\right)}{\overline{\psi}} - 1\right] \mathbb{V}\mathrm{ar}\left(\alpha_i\right) dP^{M,F} \\
& -\phi(1-\omega)\left(\epsilon^P - \phi\right)\left[(1-\overline{\alpha}) \frac{\overline{\gamma}\left(\omega\epsilon^P + \phi(1-\omega)\right)}{\overline{\psi}^2} - \frac{\overline{\alpha\gamma}}{\overline{\psi}}\right] \mathbb{C}\mathrm{ov}\left(\alpha_i, z_i\right) dP^{M,F}
\end{aligned}$$

Proof: Appendix B.4.

Proposition 4 show that when we consider the case with adjustment costs, the response can be either increasing or decreasing in firm heterogeneity depending on the sign of the brackets. The two opposing effects of firm heterogeneity that cause is this is 1) Firm heterogeneity in labor shares $1 - \alpha_i$ imply that larger firms rely less on labor, and therefor generate a lower aggregate labor response (this dampens the labor response), 2) With adjustment costs the overall output response gets amplified in the presence of firm heterogeneity (proposition 2). Thus the effect of firm heterogeneity on the labor respons depends on the parameterization of the model and the sectoral material share. Without adjustment costs, $\omega = 0$, the variance of material shares dampens the labor response. A dampening through the covariance depends on $\mathbb{C}\text{ov}\left(\alpha_i, z_i\right) > 0$ and that $1/2 < \bar{\alpha}$[2]. In our data, the covariance is positive in all except one sector. Also, 92% of firms operate in sectors where $1/2 < \bar{\alpha}$. When ω approaches unity, no changes in labor or dampening is observed. Overall, for a realistic parameterization, we expect firm heterogeneity to generate a dampening in the labor response.

3 Data and Facts About Danish Firms

The previous section established two fundamental points about the role of firm heterogeneity in shock amplification. First, firm heterogeneity only matters for the amplification of output (and prices) to the extent that real rigidities are present. Second, even without rigidities, firm heterogeneity dampens the influence of foreign shocks on labor. Critical for both these points is a positive correlation between firm size and materials expenditure as this implies that larger firms are more affected by the foreign shock and can easily adjust their output because materials are fully flexible.

In this section, we provide empirical evidence for these predictions. First, we show that large firms are indeed also more materials-intensive. Second, we illustrate that large firms respond significantly more than small firms to a foreign supply shock. Third, we find that the response of labor to a foreign supply shock is significantly lower than the response of the materials, suggesting that adjusting labor is costly in the short run.

3.1 Data

We first outline our data sources and construction. These data are used both in the empirical analysis and to calibrate the heterogeneous firm model in the next section. Detailed information is provided in Appendix A. We draw information from four Danish registers from Statistics Denmark. Firms are identified across all registers by a single firm identifier (CVRNR). We restrict attention to 1999-2017 as the registers primarily

2. This follows immediately by inserting $\bar{\psi} = \phi$ which holds true when $\omega = 0$, see Appendix B.3.

include industrial firms before 1999.

The Danish Firm Statistics Register (FirmStat) and the accounting statistics dataset (Regnskab) cover the universe of Danish private-sector firms, except agriculture, financial institutions, and public administration. We obtain the CVRNR, sector code (six-digit NACE code), number of full-time employees, sales, labor compensation, materials, and value-added reported in Danish Kroner (DKK) from these datasets. For each firm we also obtain total exports and imports across all products (service and goods trade) and countries of destination/origin. We aggregate the sector codes to the 2-digit ISIC rev. 4 to match these to the sector-level Input-Output data from Statistics Denmark.

We impose a set of restrictions on the data. First, we restrict attention to firms with positive sales and labor compensation. The resulting number of unique firms is more than 289,000 (Table A.3 in Appendix A). However, many of these firms are very small (the third quartile of the employment distribution is around 5). Therefore, we restrict attention to firms with at least five employees. The resulting dataset has more than 97,000 firms and is used for the calibration of the heterogeneous firm model. It covers around 52 percent of private value-added in national accounts, 79 percent of exports, and 74 percent of imports of goods trade (Table A.4 in Appendix A). Thus, our sample represents the national accounts fairly well.[3]

We construct our calibration sample as follows: The Danish firms are divided into different sectors. We choose to aggregate some sectors, either because the number of firms within a sector is limited or because the sector has an average markup below one, which is not compatible with firms being substitutes. We also exclude all public sectors as our sample does not cover public firms. The resulting number of sectors is 44 (these are listed in Appendix A). The total sector sales, material expenditures, labor compensation, imports, and exports are calibrated to match the IO data from Statistics Denmark. The heterogeneity and correlation in sales, import shares, export shares, and materials shares within sectors are calibrated based on our firm-level dataset. Thus, our sample aggregates to the aggregate private Danish economy, and the firm heterogeneity is calibrated based on the universe of Danish firms, which ensures that the calibrated sample in the quantitative model is as close to actual data as possible.

Our shift-share identification of the foreign supply shocks requires information on the firms' exports and imports at the product and country level. We obtain information on the firm's imports and exports at a detailed product- and destination-level from the Danish Foreign Trade Statistics Register. The dataset contains trade flows at the 8-digit Combined Nomenclature, but we aggregate up to the HS6-level to be comparable with

3. As a comparison, the sample from Dhyne et al. (2021) on Belgian firms covers 66 percent of value-added. Our data with the same sample restrictions cover 67 percent of aggregate value-added (the sample with all firms in Table A.3). Thus, we find our sample comparable to other studies.

the Baci data from CEPII used to construct the instrument (Gaulier and Zignago, 2010). As this dataset only includes goods trade, we only apply it for estimation purposes and calibrate our model using the trade flows that include service trade.

A critical transmission mechanism of supply shocks is through firm adjustments in the price-level. To investigate the pass-through of cost shocks to prices, we combine the export unit values from the Foreign Trade register with the Manufacturers' Sales of Goods database (VARS), the Danish version of the Prodcom statistics regulated by Eurostat.[4] The register contains the sales in value and volume at a detailed product-level, enabling us to construct firm-specific unit values. The resulting estimation sample contains above 24,000 firms and covers around 78% of exports and 70% of imports in aggregate goods trade from national accounts, making it representative of the firms being directly hit by foreign shocks.

Table 1 displays several characteristics of Danish firms based on their trade status. 45% of firms are only oriented to the domestic market and thus not directly affected by foreign shocks. Even so, this category only accounts for 12% of aggregate sales, and the remaining 55% trading firms account for 88% of aggregate sales. The fact that 55% of Danish firms are trading is far beyond the number of trading firms reported in other studies.[5] In Di Giovanni et al. (2024), below 20% of French firms are either importing or exporting. Similarly, only 19% (12%) of Belgian firms are importing (exporting) in Dhyne et al. (2021). These discrepancies occur for two reasons. First, we restrict the sample to firms above five employees. Without this restriction, the number of non-trading firms changes to 58% and their share of total sales to 15%. Second, and most importantly, whereas the before-mentioned papers only focus on goods trade, we consider total trade flows, i.e., including service trade. This difference increases the number of trading firms in our calibration sample by 21,687, which would incorrectly have been counted as non-trading firms. Consequently, also including service trade provides a more realistic picture of which firms that are directly affected by foreign shocks. This has the main advantage that we do not have to count service sectors as non-tradables in our general equilibrium model.

3.2 Facts About Danish Firms

In this section, we highlight three facts about Danish firms that are vital for the transmission mechanisms of foreign supply shocks and, in particular, the contribution of firm heterogeneity for shock transmission.

4. We refer to Smeets and Warzynski (2013) for a similar application on Danish data.
5. In a recent paper, Boehm et al. (2023a) find that around 40% of US firms are exporting using a novel dataset.

Table 1: Summary Statistics by Connection Type

	Firms	Average Sales	Average shares Labor	Import	Export	Share of sample Firms	Sales	Import	Export
All firms	97,481	45.534	0.485	0.065	0.079	1.000	1.000	1.000	1.000
Domestic only	43,443	11.700	0.548			0.446	0.115		
Export only	9,429	21.668	0.513		0.124	0.097	0.046		0.024
Import only	14,114	24.472	0.449	0.069		0.145	0.078	0.032	
Exporter and importer	30,495	110.861	0.403	0.175	0.215	0.313	0.762	0.968	0.976

Notes: The table displays summary statistics of the universe of Danish firms. The firms are split in different categories based on their export and import orientation. The variables are averaged over time. Sales are reported in Mio. DKK. The labor share is defined as the share of labor compensation in total firm expenditures (labor costs and material costs). The import share is defined as import relative to total firm expenditures, and the export share is defined relative to firm sales.
Source: Firm-level data are obtained from the FirmStat, Regnskab, Foreign Trade Statistics, registers from Statistics Denmark.

3.2.1 Correlation of Sales and Expenditure Shares

Fact 1: ***Within sectors, larger firms are more materials-, import-, and export-intensive.***

As shown in Section 2, firm heterogeneity amplifies the output response to foreign shocks when the variance of material shares are positive and a systematic correlation between output and material shares exists. In Figure 2, we plot the firms' sales against their materials share in total expenditures, imports share in total material expenditures and export share of total sales. For all firms, we de-mean with the sector average implying that if all firms within a sector were of identical size and had the same share of expenditures/sales, they would all be located at $(0, 0)$. Not only do the graphs illustrate that firms are heterogeneous within a sector, they also display a clear systematic correlation within sectors of the firm size and export and expenditure shares.

The systematic correlations within sectors highlight that firms of different sizes are likely affected through different transmission mechanisms. Small firms are mainly affected by shocks to labor costs, middle-size firms by their domestic supplier network, and large firms by their foreign suppliers and buyers. Thus, we expect large firms to be most severely affected by foreign shocks, which is what the second fact establishes.

3.2.2 Firm Size and Response to Foreign Shocks

Fact 2: ***Larger firms respond more to foreign supply shocks.***

A critical prediction from Section 2 is that larger firms respond more to foreign shocks. This is generated because larger firms are most affected by the shock because they apply a larger share of materials, and can easily adjust their output because they mainly rely on flexible production inputs.

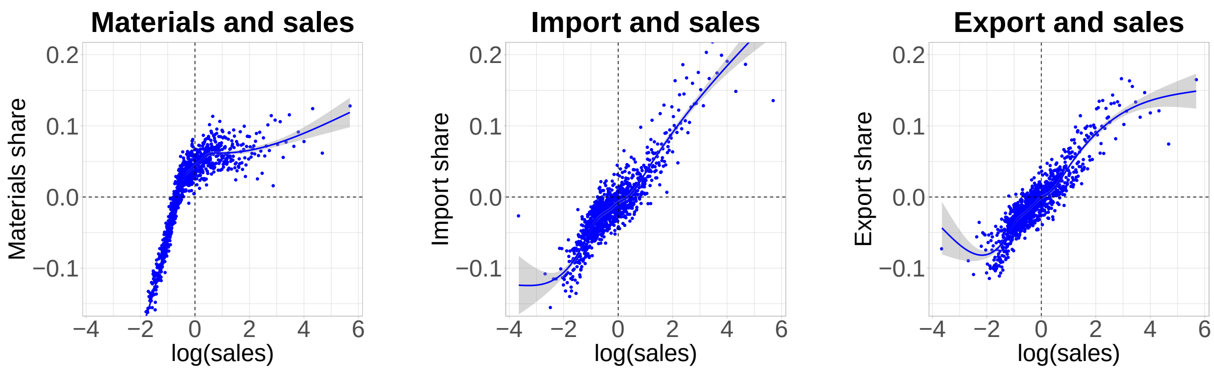


Figure 2: Firm-Level Shares and Sales

Notes: The Figure shows the material share in total expenditures, import share in total materials, and export share of sales, binned into 500 bins and ordered by their size of total sales. All variables are demeaned by the sector average. *Source:* Firm-level data are obtained from the FirmStat, Regnskab, Foreign Trade Statistics registers from Statistics Denmark.

In this section, we provide causal evidence on the dynamic heterogeneous effects of foreign supply shocks faced by Danish firms. Consistent with the partial equilibrium model in the next section, we measure a negative foreign supply shock as an exogenous increase in the firm's import price.[6] The shock is meant to reflect factors such as a negative productivity shock or supply chain disruptions that likely affect foreign production costs, giving rise to an increase in the import price met by Danish firms. We apply shift-share instruments to obtain exogenous variation in foreign supply following a long tradition in international trade (Hummels et al., 2014; Huneeus, 2018; Dhyne et al., 2021; Huneeus et al., 2021; Dhyne et al., 2022).

Firm-specific foreign supply shocks. To properly identify the effect of an exogenous foreign supply shock, we construct an instrument following Hummels et al. (2014). However, in contrast to that paper, our shift-share instrument is based on the quantity of foreign supply rather than the value. The reason is that a negative foreign supply shock is expected to decrease foreign supply and increase foreign prices. Thus, the change in the value of foreign supply is ambiguous. We construct the instrument as follows:

$$S^{shock}_{i,s,t} = \sum_{p,c} \mu^{IM}_{i,c,p,t-1} S^{EX}_{c,p,t} \tag{9}$$

The instrument is defined as a weighted average of the export quantity of country c of product p to all countries except Denmark, $S^{EX}_{c,p,t}$. The shares $\mu^{IM}_{i,c,p,t-1}$ are defined based on the Danish firm's import data and measure the share of total firm i imports originating from country c and product p. Thus, across countries and products, these shares sum to unity. In short, the shocks represent the percentage change in foreign

6. This is well in line with conventional macroeconomic supply shocks identified as shocks to oil or energy prices (Kilian, 2008).

supply that Danish firms would face if they took the rest of the world's average supply as given, averaging across markets according to the markets of relevance to them.

Following Adao et al. (2019), our setting relies on exogeneity of the shifters. Borusyak et al. (2022) outline two criteria for the consistency of the instrument. First, the shocks should be as good as randomly assigned. In our setting, this amounts to the shifters being exogenous to the individual firm. Second, the instruments should incorporate many sufficiently independent shocks, each with a relatively small exposure. Taking the year 2005 as an example, Danish importing firms operate on a total of 48,070 unique markets (combinations of HS6 codes and countries of origin). Not only does this ensure a large sample of shocks - it is also highly unlikely that they are all correlated and that individual markets dominate. Furthermore, the relevance of the instrument only holds if the individual firm is only exposed to a small number of shocks. The median number of markets a firm imports from is 8 highlighting that individual firms are only exposed to relatively few shocks. In Appendix A, we provide further details.

The underlying assumption is that supply shocks drive the foreign export supply, but we cannot entirely rule out that demand shocks may affect foreign exports.[7] As an attempt to control for demand shocks, we include a shift-share instrument of the firm's export demand. These demand shocks are constructed analogously to (9) as:

$$D^{shock}_{i,s,t} = \sum_{p,c} \mu^{EX}_{i,c,p,t-1} D^{IM}_{c,p,t} \tag{10}$$

$\mu^{EX}_{i,c,p,t-1}$ is the share of firm i's exports originating from country c product p and $D^{IM}_{c,p,t}$ is the foreign import demand from all countries except Denmark. As demand shocks are expected to influence the export demand of the firm, this is expected to control for firms being simultaneously hit by demand shocks.

Empirical specification. Let $Y_{i,s,t}$ denote some firm-level outcome of a firm in sector s. To estimate the heterogeneous dynamic effects of the supply shock, we consider the following local projection:

$$\begin{aligned} \ln Y_{i,s,t+h} = &-\beta^h_S \ln S^{shock}_{i,s,t} - \beta^h_{S,het}\omega_{i,s,t-1} \ln S^{shock}_{i,s,t} \\ &+ \lambda^h_D \ln D^{shock}_{i,s,t} + \boldsymbol{\kappa}^h \boldsymbol{X}_{i,s,t-1} + \delta^h_i + \delta^h_s \times \delta^h_t + \varepsilon_{i,s,t+h} \end{aligned} \tag{11}$$

where $S^{shock}_{i,s,t}, D^{shock}_{i,s,t}$ are our supply and demand shocks from above, included to control for correlated demand and supply shocks, $\boldsymbol{X}_{i,s,t-1}$ is a set of controls (two lags of both

7. As we in the quantitative model measure a negative foreign supply shock as an increase in the import price, whether the increase is driven by supply or demand shocks does not per se invalidate our estimations. Further, a negative foreign demand shock is a priori expected to *decrease* the import price rather than, as we observe, *increase* the import price.

shocks, two lags of the dependent variable[8]), δ_i^h is a firm fixed effect, and $\delta_s^h \times \delta_t^h$ is a sector-time fixed effect included to control for GE-effects.

To evaluate heterogeneous responses of the supply shock across the firm size distribution we also include an interaction term $\omega_{i,s,t-1} \times \ln S_{i,s,t}^{shock}$ which interacts with the supply shock with a weight $\omega_{i,s,t-1} = \ln z_{i,s,t-1} - \ln \bar{z}$, where $\bar{z}$ denotes average sales across all firms. This implies that β_S^h captures the average effect of the supply shock at horizon h, whereas $\beta_S^h + \beta_{S,het}^h \times (\ln z_{i,s,t-1} - \ln \bar{z})$ is the effect for a firm with size $z_{i,s,t-1}$ in the period before the shock. Thus, the coefficients $\{\beta_S^h\}_{h=0}^H, \left\{\beta_{S,het}^h\right\}_{h=0}^H$, are interpreted as capturing the partial equilibrium response to foreign supply shocks. We scale all coefficients such that it corresponds to a 10% increase in the import price on impact.

Estimation results. Figure 3 displays the responses to a temporary foreign supply shock, scaled to deliver a 10% increase in the import price on impact. For the median firm, we find that the profile of the import price is persistent and lasts around two years into the future. The increase in import prices gives rise to an increase in the firm's price level, corresponding to around 3%. This indicates that some of the cost-push shock is passed onto consumers and other firms. As the average import share in total expenditures (labor and materials) in the sample is 32%, this corresponds to a close to 100% pass-through of cost shocks to prices. Thus, the estimated pass-through is above those estimated in Amiti et al. (2019) on 60%. However, the price response is estimated with large uncertainty, reflected by the wide confidence bands.[9] The cost shock to the firm implies a significant drop in real sales of around 6% on impact, and the decline persists for at least two years.

The estimated impulse responses imply that foreign supply shocks have significant and persistent effects on domestic firms who import, manifesting in reduced production and higher prices. Though one might expect a large drop in employment following an adverse foreign supply shock, we only estimate a small decrease. Here, it is important to bear in mind that we estimate a *partial equilibrium* response and that the *general equilibrium* outcome of such a shock might be a larger decrease in employment.[10]

Diving into the heterogeneous effects, in Figure 3, we display the effect of the first

8. The use of lags of the left-hand side variable is standard in the LP literature to improve inference as well as ensuring stationary, see Montiel Olea and Plagborg-Møller, 2021; Durante et al., 2022; Drechsel, 2022.

9. The fact that we estimate a large effect on prices following a supply shock is consistent with recent research highlighting that prices are more flexible with respect to supply shocks (Bunn et al. (2022), Phelan and L'Huillier (2023)).

10. Meier and Pinto (2020) estimates the effect of the COVID-19 disruption in China, which significantly raised import prices, and finds a large reduction in employment across US sectors in a framework that also includes general equilibrium effects.

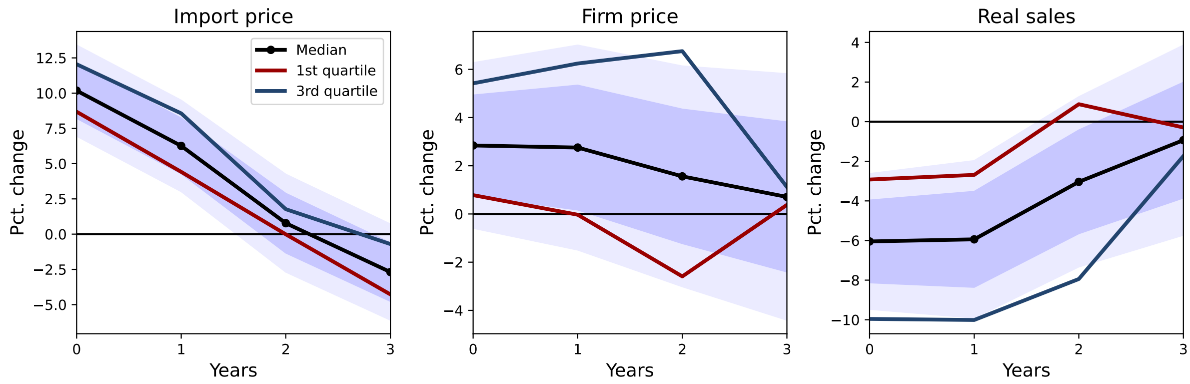


Figure 3: Heterogeneous impulse-responses to negative foreign supply shock

Notes: The Figure shows the dynamic impulse responses on several firm-level outcomes from a foreign supply shock, scaled to deliver a 10% increase in the import price. The responses are shown for the median firm in terms of log sales and the 1st and 3rd quartile of the firm size distribution. The standard errors are clustered at the sector-time level. 66 and 90 percent confidence intervals are reported as the shaded blue areas. *Source:* Firm-level data are obtained from the FirmStat, Regnskab, Foreign Trade Statistics Register, and VARS registers from Statistics Denmark.

and third quartile of the log sales distribution. Reassuringly, we find no indication that the firm size distribution affects the influence of the foreign shock on the import price. This implies that all firms experience a similar magnitude of the shock regarding changes in the import price. Thus, it seems reasonable in our partial equilibrium model to shock all firms with the same shock, irrespective of differences in size. Our results confirm the main mechanisms in partial equilibrium we highlight in this paper: larger firms are most severely affected by foreign supply shocks. Firms at the third quartile increase their price on impact by 5% and decrease sales by 10%, compared to 1% and 3% for the firms at the first quartile.

In Appendix A.3, we display several robustness checks, such as changing the number of lags and changing the estimation sample. Overall, we find our results robust to these changes.

3.2.3 Response of Labor and Materials to Foreign Shocks

Fact 3: ***Labor is less responsive than materials to foreign shocks, suggesting that labor is more costly to adjust.***

Proposition 2 established that a necessary condition for shock amplification is that adjustment costs of labor are present. To establish that it is more costly to adjust labor than materials, we estimate the impact of a foreign supply shock on those variables. We apply the shift-share instrument and the estimation framework outlined in equation (11) but leave out the interaction term with firm size ($\omega_{i,s,t-1}$) as the adjustment cost in our model is invariant to firm size.

Following a foreign supply shock corresponding to a 10% increase in the import price, firms reduce on impact their materials expenditures by around 5%, corresponding well with the drop in output (Figure 4). The response of labor is less than half that size, but way more persistent. Indeed, the response in labor expenditures a period after the shock is even larger than the on-impact effect of the shock. This suggests that adjusting labor is costly and sluggish.[11] In the next section, we apply these empirical impulse responses to estimate the adjustment cost of labor and materials in our model. We find that the adjustment cost of labor is significantly above the adjustment cost of materials, confirming our third stylized fact.

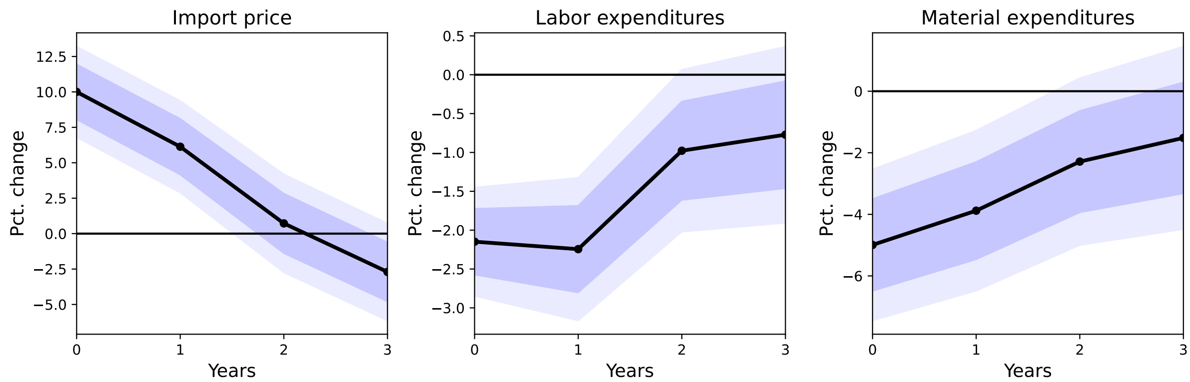


Figure 4: Impulse-Responses to Negative Foreign Supply Shock

Notes: The Figure shows the dynamic impulse responses on labor and materials from a foreign supply shock, scaled to deliver a 10% increase in the import price. The standard errors are clustered at the sector-time level. 66 and 90 percent confidence intervals are reported as the shaded blue areas. *Source:* Firm-level data are obtained from the FirmStat, Regnskab, Foreign Trade Statistics Register, and VARS registers from Statistics Denmark.

4 Quantitative Multi-Sector Heterogeneous Firm Model

So far, we have established in a stylized heterogeneous firm model and a set of empirical stylized facts, that firm heterogeneity combined with adjustment costs generates an amplification in the output response to foreign shocks, but dampening in the labor response. In this section, we lay out a New-Keynesian multi-sector heterogeneous firm model with adjustment costs, building on the stylized model in section 2. The model is closely calibrated to match the empirical stylized facts and the universe of Danish firms, thus rendering it representative of the Danish economy.

The model is a discrete-time general equilibrium model featuring both sectoral and firm heterogeneity. One period corresponds to one quarter. The economy is inhab-

11. Another interpretation is that the elasticity of substitution between labor and materials is sufficiently high. If we are to reproduce the IRFs in Figure 4) in our theoretical model presented in Section 4 using a higher elasticity of substitution instead of adjustment costs we require it be in the range of 3-6, which is far beyond short-run empirical estimates (Oberfield and Raval, 2021) and the convention of an elasticity close to unity used in most models (Huo et al. (2023), among others). Also, significant adjustment costs of labor were still present, implying that the main mechanisms are still operative in this setting.

ited by $\mathcal{S}$ sectors, corresponding to the 44 sectors we used in our main data sample in Section 3. Each sector is inhabited by a continuum of firms of measure 1. We allow for 4 types of firm heterogeneity, namely heterogeneous firm size, materials shares, import and export shares.[12] The distribution of firms within each sector is a 4-dimensional multivariate distribution, and for simplicity, we take this distribution to be time-invariant, i.e., the model features permanent type heterogeneity. This multivariate distribution allows for systematic correlations between our 4 dimensions of heterogeneity, and this is the main departure from the standard multi-sector firm model presented in e.g. Long Jr and Plosser (1983), Carvalho (2008), and Huo et al. (2023).

4.1 Quantitative Firm Model

Production. In period t, firm i in sector s produce output, $z_{i,s,t}$, using a CES technology over labor and intermediate goods:

$$z_{i,s,t} = \Gamma_{i,s}\left[\alpha_{i,s}^{\frac{1}{\phi}} m_{i,s,t}^{\frac{\phi-1}{\phi}} + (1-\alpha_{i,s})^{\frac{1}{\phi}}\,\ell_{i,s,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{12}$$

where $0 \leq \alpha_{i,s} \leq 1$ is a firm-specific material expenditure share and $\phi > 0$ is the elasticity of substitution between materials and labor. Labor is rented from households at the going sector wage rate $W_{s,t}$, which is taken as given by individual firms. The input of materials, $m_{i,s,t}$, is a CES aggregate of imported materials and domestic materials:

$$m_{i,s,t} = \left[\gamma_{i,s}^{\frac{1}{\vartheta}}\left(m_{i,s,t}^{F}\right)^{\frac{\vartheta-1}{\vartheta}} + (1-\gamma_{i,s})^{\frac{1}{\vartheta}}\left(m_{i,s,t}^{D}\right)^{\frac{\vartheta-1}{\vartheta}}\right]^{\frac{\vartheta}{\vartheta-1}} \tag{13}$$

where $0 \leq \gamma_{i,s} \leq 1$ is a firm-specific import share. The firm-specific material price is:

$$P_{i,s,t}^{M} = \left[\gamma_{i,s}\left(P_{s,t}^{M,F}\right)^{1-\vartheta} + (1-\gamma_{i,s})\left(P_{s,t}^{M,D}\right)^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}}$$

Note that all firms within a sector face the same prices of foreign and domestic materials $P_{s,t}^{M,F}, P_{s,t}^{M,D}$, but the overall material price $P_{i,s,t}^{M}$ is firm-specific because of heterogeneity in import shares. The input of domestic materials, $m_{i,s,t}^{D}$, is an aggregate of materials from all other domestic sectors:

$$m_{i,s,t}^{D} = \left[\sum_{j\in\mathcal{S}} \Theta_{s,j}^{\frac{1}{\eta}} s_{j,i,s,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{14}$$

12. We impose heterogeneity in firm size as measured by output $z_{i,s}$ directly in the model, which is made possible by the assumption of constant returns to scale in production (see eq. (12)). An earlier version of the paper had heterogeneity in firm size coming from a combination of productivity and decreasing returns to scale. The results are unchanged.

where Θ is a $\mathcal{S} \times \mathcal{S}$ domestic IO-matrix satisfying $\sum_j \Theta_{s,j} = 1$, where element s, j encodes the share of materials brought by sector s from sector j. The implied sectoral price of domestic materials is:

$$P_{s,t}^{M,D} = \left[\sum_{j \in \mathcal{S}} \Theta_{s,j} \left(P_{j,t} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}, \tag{15}$$

Adjustment costs. To make the amplifying effect of firm heterogeneity and real rigidities from Section 2 operative in our quantitative model, we allow for adjustment cost on labor and materials. We assume firms have to pay costs $\theta_{i,s,t}^{\ell} \left(\ell_{i,s,t}, \ell_{i,s,t-1} \right), \theta_{i,s,t}^{m} \left(m_{i,s,t}, m_{i,s,t-1} \right)$, and we take the forms of adjustment costs to be standard quadratic:

$$\theta_{i,s,t}^{\ell} = \frac{\theta^{\ell}}{2} \left(\frac{\ell_{i,s,t}}{\ell_{i,s,t-1}} - 1 \right)^2 \ell_{i,s,t-1}, \quad \theta_{i,s,t}^{m} = \frac{\theta^{M}}{2} \left(\frac{m_{i,s,t}}{m_{i,s,t-1}} - 1 \right)^2 m_{i,s,t-1}$$

Sales and price setting. Firms in sector s sell their products in a common market, competing against all other firms in that sector. We introduce heterogeneity in sales destinations by allowing for a time-varying heterogeneous demand shifter. Firms face the following CES demand function:

$$z_{i,s,t} = \varrho_{i,s,t} \left(\frac{p_{i,s,t}}{P_{s,t}} \right)^{-\epsilon^P} Z_{s,t} \tag{16}$$

where $Z_{s,t}$ is aggregate demand for sector s goods, $P_{s,t}$ is the aggregate sectors price index, $p_{i,s,t}$ is the price set by firm i in sector s, and $\epsilon^P > 1$ is the elasticity of substitution between firm-level products within a sector. The firm-specific demand shifter is given by:

$$\varrho_{i,s,t} = \frac{\xi_{i,s}^{X} Z_{s,t}^{X} + \xi_{i,s}^{D} Z_{s,t}^{D}}{Z_{s,t}} \tag{17}$$

and $Z_{s,t}^X, Z_{s,t}^D$ are measures of foreign and domestic demand, satisfying $Z_{s,t}^X + Z_{s,t}^D = Z_{s,t}$ in equilibrium. The heterogeneous weights $\xi_{i,s}^X = \xi_{i,s} \frac{z_{i,s}}{Z_s^X}$ and $\xi_{i,s}^D = (1 - \xi_{i,s}) \frac{z_{i,s}}{Z_s^D}$ are the firm's steady-state share of foreign and domestic demand. These define the firm's exposure to foreign and domestic demand observed in the data. Profits are given by:

$$\Pi_{i,s,t} = p_{i,s,t} z_{i,s,t} - w_{s,t} \ell_{i,s,t} - p_{i,s,t}^{M} m_{i,s,t} - \theta_{i,s,t}^{\ell} - \theta_{i,s,t}^{m} - F_s \tag{18}$$

where the latter 2 terms are the various adjustment costs, and F_s is a sector-specific fixed cost, calibrated to match the sectoral profits from the IO-tables. The problem of the firm is to pick $\left\{p_{i,s,t}, z_{i,s,t}, \ell_{i,s,t}, m_{i,s,t}, m^D_{i,s,t}, m^F_{i,s,t}\right\}_{t=0}^{\infty}$ so as to maximize the present discounted value of profits (18) subject to constraints (12)-(16). We assume that firms discount using $\beta = \frac{1}{1+r}$, where r is the steady state real interest rate.

Aggregation. Total sectoral production is given by the CES aggregate:

$$Z_{s,t} = \left[\int \left(\varrho_{i,s,t}\right)^{\frac{1}{\epsilon^P}} \left(z_{i,s,t}\right)^{\frac{\epsilon^P-1}{\epsilon^P}} \mathrm{d}i\right]^{\frac{\epsilon^P}{\epsilon^P-1}}$$

with associated price indices:

$$P_{s,t} = \left[\int \varrho_{i,s,t} \left(p_{i,s,t}\right)^{1-\epsilon^P} \mathrm{d}i\right]^{\frac{1}{1-\epsilon^P}}$$

Capital. As in, among others, Ho et al. (2022) and Di Giovanni et al. (2024), our model does not include capital. From a data perspective, Danish firm-level capital data is limited and often imputed for firms with below 50 employees. Limiting our analysis to only firms with above 50 employees would severely reduce the number of firms and create an unrepresentative description of the firm distribution. From a quantitative perspective, Huo et al. (2023) shows in a multi-sector multi-country model with four types of shocks (labor supply, productivity, intermediate inputs, and investments) that excluding the investment shocks only reduces the average business cycle correlation by 10%.

4.1.1 Modelling Micro-Level Heterogeneity

Our heterogeneous firm model features 4 distinct sources of heterogeneity: Heterogeneity in firm size ($z_{i,s}$), material intensiveness ($\alpha_{i,s}$), export intensiveness ($\xi_{i,s}$), and import intensiveness ($\gamma_{i,s}$). Here we show how we pick functional forms for the variables. For firm size $z_{i,s}$ we opt for a flexible modeling, namely a mix of a log-normal and a Pareto distribution:

$$\log z_{i,s} \sim \begin{cases} \mu^z_s \cdot \mathcal{N}\left(0, \left(\sigma^z_s\right)^2\right), & z_{i,s} < \bar{z}_s \\ \mu^z_s \cdot \text{Pareto}\left(a^z_s\right), & z_{i,s} \geq \bar{z}_s \end{cases}$$

That is, the distribution is log-normal below some threshold $\bar{z}_s$ and Pareto above the threshold, consistent with empirical observations that the Pareto distribution only fits the firm size distribution well at the top of the distribution. (Combes et al., 2012). We fix $\bar{z}_s$ at the 80th percentile of the firm size distribution and calibrate $\mu^z_s, \sigma^z_s, a^z_s$ to match the mean, variance and skewness of the firm size distribution from the data.

From section 3, we know firm-level use of materials, imports, and exports correlate strongly with firm size. We wish to replicate this in the model, i.e. generate arbitrary correlations between $\alpha_{i,s}, \xi_{i,s}, \gamma_{i,s}$ and $z_{i,s}$. We do this by using a Gaussian copula, ensuring that the parameters lie in the appropriate interval $[0, 1]$. In particular, we assume:

$$\begin{bmatrix} \alpha_{i,s} \\ \gamma_{i,s}^{\gamma>0} \\ \xi_{i,s}^{\xi>0} \end{bmatrix} = F_{\beta}^{-1}\left(\Phi\left(\boldsymbol{\rho}_s^z \ln z_{i,s} + \varepsilon_{i,s}\right), \boldsymbol{a}_s, \boldsymbol{b}_s\right)$$

where F_β is the CDF of the beta distribution (and F_β^{-1} hence the quantile function), $\varepsilon_{i,s}$ is a standard normal noise term and $\boldsymbol{\rho}_s^z, \boldsymbol{a}_s, \boldsymbol{b}_s$ are vectors of parameters. For instance, $\boldsymbol{\rho}_s^z = \left[\rho_s(z,\alpha), \rho_s\left(z,\gamma^{\gamma>0}\right), \rho_s\left(z,\xi^{\xi>0}\right)\right]'$ captures the correlation between the various shares and log firm size within a given sector s. Similarly the vectors $\boldsymbol{a}_s, \boldsymbol{b}_s$ characterize the beta distribution and determine the mean and variance of $\alpha_{i,s}, \gamma_{i,s}^{\gamma>0}, \xi_{i,s}^{\xi>0}$. Let $D_{i,s}^{\gamma}, D_{i,s}^{\xi}$ denote dummies for whether firms import or export respectively. We assume that firms do not export nor import according to the following flexible rule:

$$D_{i,s}^{\xi,\gamma} = \begin{cases} 0, & \ln z_{i,s} + \varepsilon \cdot \sigma_{\gamma,\xi} < \tau_{\gamma,\xi} \\ 1, & \text{else} \end{cases}$$

where ε is standard normal innovation and $\sigma_{\gamma,\xi}, \tau_{\gamma,\xi}$ are parameters. Given a small variance of the innovation $\sigma_{\gamma,\xi}$ and a high threshold $\tau_{\gamma,\xi}$ this formulation implies that only the larger and more productive firms will export and import. The reverse scenario implies no relationship between size and trade. Thus, this formulation can produce observations similar to those made by the model in Melitz (2003) where only the most productive firms overcome the fixed cost of selling to foreign markets and thus trade. We use a similar rule for firms that only export or import:

$$D_{i,s}^{\xi} = \begin{cases} 0, & \ln z_{i,s} + \varepsilon \cdot \sigma_{\xi} < \tau_{\xi} \\ 1, & \text{else} \end{cases}$$

$$D_{i,s}^{\gamma} = \begin{cases} 0, & \ln z_{i,s} + \varepsilon \cdot \sigma_{\gamma} < \tau_{\gamma} \\ 1, & \text{else} \end{cases}$$

The realized level of import and export intensities are:

$$\gamma_{i,s} = \gamma_{i,s}^{\gamma>0} \cdot D_{i,s}^{\xi,\gamma} \cdot D_{i,s}^{\gamma}$$
$$\xi_{i,s} = \xi_{i,s}^{\xi>0} \cdot D_{i,s}^{\xi,\gamma} \cdot D_{i,s}^{\xi}$$

This completes the description of the heterogeneous firm model.

4.2 General Equilibrium Model

This section presents the remaining parts that make up our general equilibrium model. We aim to keep the rest of the model as standard as possible, given our extensive supply side.

4.2.1 Households

The domestic economy is inhabited by a representative household that derive utility from a consumption bundle C and disutility from labor supply:

$$U = \sum_{t=0}^{\infty} \beta^t \left\{ u\left(C_t\right) - \sum_{s \in \mathcal{S}} G_s\left(L_{s,t}\right) \right\} \tag{19}$$

with functional forms $u\left(C_t\right) = \ln\left(C_t\right), G_s\left(L_{s,t}\right) = \frac{\vartheta_s}{1+\frac{1}{\nu}} L_{s,t}^{1+\frac{1}{\nu}}$. The budget constraint is given by:

$$C_t + A_t = \left(1 + r_t\right) A_{t-1} + \sum_{s \in \mathcal{S}} \left(w_{s,t} L_{s,t} + \Pi_{s,t}\right) \tag{20}$$

where A_t is the level of domestic assets and r_t is the real return measured in units of the domestic CPI P_t. Optimization implies the standard Euler equation:

$$u'\left(C_t\right) = \beta\left(1 + r_{t+1}\right) u'\left(C_{t+1}\right) \tag{21}$$

Labor supply. Following an abundant literature, we assume that labor supply is set at the union level, and subject to adjustment costs generating nominal wages stickiness (Erceg et al., 2000; Schmitt-Grohé and Uribe, 2005). We assume sector specific unions, implying that for each sector s we obtain a wage setting curve:

$$\pi_{s,t}^W = \kappa^W \left\{ \frac{G'\left(L_{s,t}\right)}{\frac{1}{\mu_s^W} \frac{W_{s,t}}{P_t} u'(C_t)} - 1 \right\} + \beta \pi_{s,t+1}^W \tag{22}$$

where $\pi_{s,t}^W = W_{s,t} / W_{s,t-1} - 1$, $u'(C_t)$ denotes the aggregate marginal utility of consumption, μ_s^W is a wage markup, and κ^W determines the degree of sticky wages. If we assume that the underlying nominal friction is á la Calvo, where unions only update wages with probability $1 - \theta^W$ each quarter, then we have $\kappa^W = \frac{\left(1-\beta\theta^W\right)\left(1-\theta^W\right)}{\theta^W}$.

Consumption goods. The consumption bundle C_t is a CES over domestic and foreign goods:

$$C_t = \left[HB^{\frac{1}{\varepsilon^C_{H,F}}} C_{H,t}^{\frac{\varepsilon^C_{H,F}-1}{\varepsilon^C_{H,F}}} + (1-HB)^{\frac{1}{\varepsilon^C_{H,F}}} C_{F,t}^{\frac{\varepsilon^C_{H,F}-1}{\varepsilon^C_{H,F}}} \right]^{\frac{\varepsilon^C_{H,F}}{\varepsilon^C_{H,F}-1}}, \tag{23}$$

where $C_{H,t}$ is consumption of domestic goods, $C_{F,t}$ is consumption of foreign goods and HB measures the degree of home-bias. The demand for the respective goods is given by:

$$C_{H,t} = HB \left(\frac{P_{H,t}}{P_t} \right)^{-\varepsilon^C_{H,F}} C_t \tag{24}$$

$$C_{F,t} = (1-HB) \left(\frac{P_{F,t}}{P_t} \right)^{-\varepsilon^C_{H,F}} C_t \tag{25}$$

where P_t is the domestic consumer price index:

$$P_t = \left[HB \cdot P_{H,t}^{1-\varepsilon^C_{H,F}} + (1-HB) \cdot P_{F,t}^{1-\varepsilon^C_{H,F}} \right]^{\frac{1}{1-\varepsilon^C_{H,F}}} \tag{26}$$

and $P_{H,t}, P_{F,t}$ are the prices of $C_{H,t}, C_{F,t}$ respectively. Consumption of home goods $C_{H,t}$ is a CES composite of goods produced across domestic sectors:

$$C_{H,t} = \left[\sum_{s \in \mathcal{S}} \Phi_s^{\frac{1}{\varepsilon^C_S}} C_{H,s,t}^{\frac{\varepsilon^C_S - 1}{\varepsilon^C_S}} \right]^{\frac{\varepsilon^C_S}{\varepsilon^C_S - 1}}, \tag{27}$$

implying the following demand curve:

$$C_{H,s,t} = \Phi_s \left(\frac{P_{s,t}}{P_{H,t}} \right)^{-\varepsilon^C_S} C_{H,t} \tag{28}$$

The domestic price $P_{H,t}$ is a CES price index of the sectoral output prices:

$$P_{H,t} = \left[\sum_{s \in \mathcal{S}} \Phi_s P_{s,t}^{1-\varepsilon^C_S} \right]^{\frac{1}{1-\varepsilon^C_S}}. \tag{29}$$

We model exports to the foreign economy using a nested formulation. At the top of the nest overall exports are determined in an Armington relation:

$$C^*_{X,t} = \left(\frac{P^*_{X,t}}{P^*_{CPI,t}}\right)^{-\varepsilon^*_X} C^*_t \tag{30}$$

where $P^*_{X,t} = \sum_s \psi^*_s \frac{P_{s,t}}{E_t}, P^*_{CPI,t}$ is the aggregate price of domestic goods in foreign currency units and the foreign CPI in foreign currency respectively. Sectoral exports are then determined as:

$$C^*_{s,t} = \psi^*_s \left(\frac{P_{s,t}/E_t}{P^*_{X,t}}\right)^{-\varepsilon^C_S} C^*_{X,t} \tag{31}$$

4.2.2 Capital Flows and International Pricing

We assume that the law of one price holds. This implies that the price of domestic goods $P_{s,t}$ in domestic currency is related to the price of domestic goods in foreign currency units $P^*_{s,t}$ as:

$$P^*_{s,t} = \frac{P_{s,t}}{E_t} \tag{32}$$

where E_t denotes the nominal exchange rate. A similar relation holds for the price of foreign goods:

$$P_{F,t} = E_t P^*_{F,t} \tag{33}$$

Free capital flows implies that returns to capital must be equalized across countries, implying that uncovered interest parity must hold:

$$(1 + i_t) = (1 + i^*_t)\,\frac{E_{t+1}}{E_t} \tag{34}$$

where i^*_t is the foreign nominal rate, which is exogenous to the domestic economy since we consider a small open economy.

4.2.3 Monetary Policy

We have calibrated the model to match the overall characteristics of the Danish economy. Given that Denmark have a fixed exchange rate towards the majority of its trading partners we opt for a fixed exchange rate regime in the baseline model, i.e.[13]:

$$E_t = \overline{E}$$

where $\overline{E}$ is the exogenous exchange rate level in the foreign economy. In a robustness check we also consider the case where domestic monetary policy operates under a Taylor rule:

$$i_t = \bar{i} + \phi^{\pi} \pi_t \tag{35}$$

where $\bar{i}$ is the target interest rate of the central bank and π_t denotes CPI inflation.

4.2.4 Market Clearing

The aggregate market clearing conditions for sector s is:

$$Z_{s,t} = C_{H,s,t} + C^*_{s,t} + R_{s,t} \tag{36}$$

where $R_{s,t} = \sum_{j \in S} \int s_{s,i,j,t} \, \mathrm{d}i$ is the total demand for materials from sector s by domestic firms. Given free capital flows domestic savings A need not equal the domestic supply of liquidity, which is zero in our model. Instead the current account identity must hold:

$$NFA_t - (1 + r_t) \, NFA_{t-1} = NX_t \tag{37}$$

where $NFA_t = A_t$ and net exports are defined as:

$$NX_t = \frac{P_{H,t}}{P_t} C^*_{H,t} - \left(\frac{P_{F,t}}{P_t} C_{F,t} + \sum_{s \in S} \frac{P^{M,F}_{s,t}}{P_t} M_{s,t} \right) \tag{38}$$

Sectoral labor market clearing requires that aggregate labor supplied to sector s equals the total demand for labor within that sector:

$$L_{s,t} = \int \ell_{s,i,t} \, \mathrm{d}i \tag{39}$$

Lastly, we define aggregate GDP as the sum of nominal sectoral value added deflated by the CPI:

$$Y_t = \sum_{s \in \mathcal{S}} \frac{PY_{s,t}}{P_t} \tag{40}$$

13. In the sample of OECD countries used in section A, 59% of Danish trade is oriented toward EUR countries, and the remainder 41% countries with a flexible exchange rate relative to the EUR. Thus, Denmark is in the middle ground between a flexible and a fixed exchange rate.

where sectoral nominal value added is $PY_{s,t} = \int p_{i,s,t} z_{i,s,t} - P^{M}_{i,s,t} m_{i,s,t} \, \mathrm{d}i$.

4.3 Calibration

We here detail the calibration of the full general equilibrium model. We solve the model by linearizing in sequence-space (Auclert et al., 2021), see Section D in the appendix.

Heterogeneous firm model. We proceed in a 2-step calibration of the firm model. We first externally fix a subset of parameters, primarily elasticities, that are identified in the existing literature. The elasticity of substitution between foreign and domestic materials θ is set to 0.5 in accordance with the short-run estimate from Boehm et al. (2019). In addition, we set the elasticity of substitution between different domestic materials $\eta = 0.2$ as estimated in Atalay (2017) and Cravino and Sotelo (2019).

We then proceed to internally calibrate the parameters governing the means, dispersion, and correlations of materials expenditure shares, import/export shares, output, and markups. The targets associated with each parameter are displayed in Table A.5 and the sector-level moments are displayed in Table A.1 and the firm-level moments in Table A.2. We minimize a quadratic loss function that contains all these targets w.r.t the parameters. Afterward, we use an exact root-finder to match all aggregate sectoral flows, ensuring consistency with national accounts. In this step we also calibrate the sectoral fixed cost F_s to ensure that we match sectoral profits in the national accounts.

We display the fit of the model against the empirical targets in Figure A.11. Given the ambitious calibration - we target 748 data moments - the fit is very good with an R^2 of between 0.988 and 0.999, depending on the targets. Note that we cannot completely match all moments because the underlying parameters affect several moments simultaneously.

GE model. The remaining parameters that need to be specified are those that relate to households and the central bank, as well as the various exogenous share parameters. We calibrate all share parameters to be consistent with sectoral national accounts data for Denmark in a specific base year (2005). We calibrate the aggregate level of home bias in consumption HB to be consistent with the level of exports implied by our heterogeneous firm model as well as an assumption of a zero net-foreign asset position in a steady state. We calibrate the entries in the IO matrix $\{\Theta_{s,j}\}$ as well as sectoral consumption demand $\{\Phi_s\}$ to the sectoral input-output table for Denmark in 2005. Lastly, the share parameters that determine demand for sectoral exports $\{\psi^*_s\}$ are calculated to match sectoral exports from the data.

Table 2: Heterogeneous Firm Model Calibration

Parameter	Meaning	Value	Target
		Externally calibrated	
θ	EoS between m^F, m^D	0.5	Boehm et al. (2019)
η	EoS between m^D	0.2	Atalay (2017)
		Internally fitted	
μ_s^z	Mean of log $z_{i,s}$ dist.	-	Aggregate sectoral output
σ_s^z	Variance of log $z_{i,s}$	-	Within sector variance of log output
a_s^z	Parameter of Pareto dist.	-	Skewness of sectoral log output
a_s^α	Location of β-distribution	-	Aggregate sectoral labor share
b_s^α	Shape of β-distribution	-	Within sector variance of labor share
a_s^γ	Location of β-distribution	-	Aggregate sectoral import share in intermediates
b_s^γ	Shape of β-distribution	-	Within sector variance of import share in intermediates
a_s^ξ	Location of β-distribution	-	Aggregate sectoral export share
b_s^ξ	Shape of β-distribution	-	Within sector variance of export shares
$\rho_s(z, \alpha)$	z, α correlation	-	Within sector correlation of log output and labor share
$\rho_s(z, \gamma)$	z, γ correlation	-	Within sector correlation of log output and import shares
$\rho_s(z, \xi)$	z, ξ correlation	-	Within sector correlation of log output and export shares
$\tau_{\gamma,\xi}$	Threshold for $\xi = 0$ and $\gamma = 0$	-	Share of firms with that do not trade
τ_γ	Threshold for $\gamma = 0$	-	Share of firms with zero imports
τ_ξ	Threshold for $\xi = 0$	-	Share of firms with zero exports
$\sigma_{\gamma,\xi}$	-	-	-
σ_γ	-	-	Share of sectoral output for importing firms
σ_ξ	-	-	Share of sectoral output for exporting firms

Notes: This table summarizes the parameters and data targets used in the quantitative model. For the markup the table reports the median and the 10th and 90th percentiles across sectors in brackets, weighted by firm level sales.

Table 3 displays the externally calibrated value of the relevant parameters. We fix the elasticity of substitution between domestic and foreign varieties in the consumption bundle of domestic households to 1.5, and use the same value for the Armington export elasticity in eq. (30). We set the elasticity of substitution between different domestic consumption goods, η^C, to 0.5 following Cravino and Sotelo (2019). We assume a unitary Frisch elasticity of labor supply, which is a standard value in the literature and close to estimates in Chetty et al. (2011) and Huo et al. (2023). The wage markup is fixed at 1.1, which is a standard value in the literature. Given that our supply side does not feature any nominal rigidities, our only source of nominal frictions is wage stickiness. We set the slope of the NKWPC to 0.03 following Auclert et al. (2023). Finally, in the version of our model with a floating exchange rate and a Taylor rule we assume that the central bank responds to CPI inflation with a coefficient of 1.5, $\phi^\pi = 1.5$.

Steady state firm distribution. Figure 5 plots material, import- and export-shares against firm size (all demeaned with sector averages). The model reproduces the empirical facts from Section 3: Larger firms are more material-intensive and trade more.

Figure 6 shows the distribution of firm size, grouped by whether firms import, export or do not trade at all. Firms that only interact with the domestic market tend to be smaller, whereas the distributions for both importing and exporting firms have

Table 3: GE Model Calibration

Parameter	symbol	Value/Target	Source
Discount factor	β	0.99	Annual interest rate = 4%
EoS C_H, C_F	$\varepsilon^C_{H,F}$	1.5	Feenstra et al. (2018) and Boehm et al. (2023b)
EoS between sectors ($C_{H,s}, C^*_s$)	ε^C_S	0.5	Cravino and Sotelo (2019)
Armington elasticity	ε^*_X	1.5	Feenstra et al. (2018) and Boehm et al. (2023b)
Frisch elasticity	ν	1	Standard
Wage markup	μ^W	1.1	Standard
Wage rigidity	κ^W	0.03	Auclert et al. (2023)
CB response to π	ϕ^π	1.5	Standard

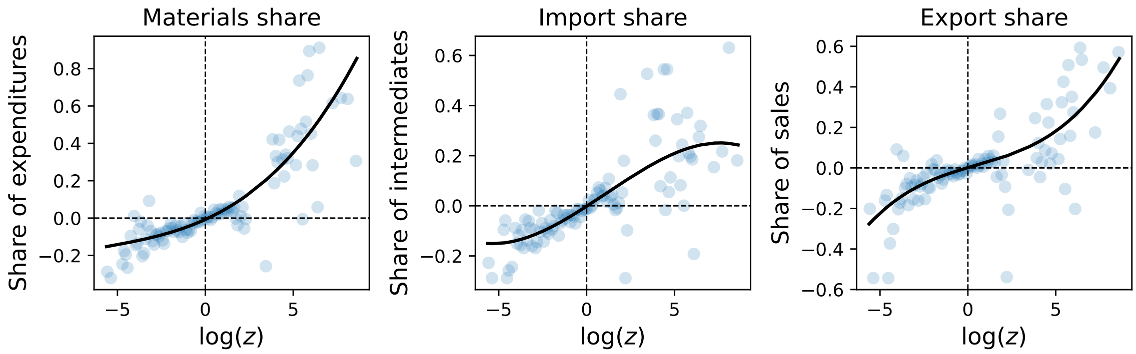


Figure 5: Firm Size Characteristics

fatter right-side tails.[14]

4.4 Impulse-Response Function Matching

Given the calibration in the prior section, we proceed to estimate the so-far unspecified parameters in the firm model using impulse-response matching, namely the adjustment costs, the materials-labor substitution elasticity, and the elasticity of substitution between firms. We follow recent macroeconomic literature by matching the average response in the model to causal evidence on foreign shocks (Nakamura and Steinsson, 2018).

We subject the model to a partial equilibrium foreign supply shock resembling a shock to import prices, keeping all other aggregates fixed. This shock corresponds to the exogenous shock considered in Section 3, except we only target the average effect. This suffices to make the model IRFs comparable to empirical estimates, which

14. The distributions for firms that import and export are bimodal due to the way we specified the underlying processes for firm output (a mix of a normal and Pareto distribution). This is not central to our results.

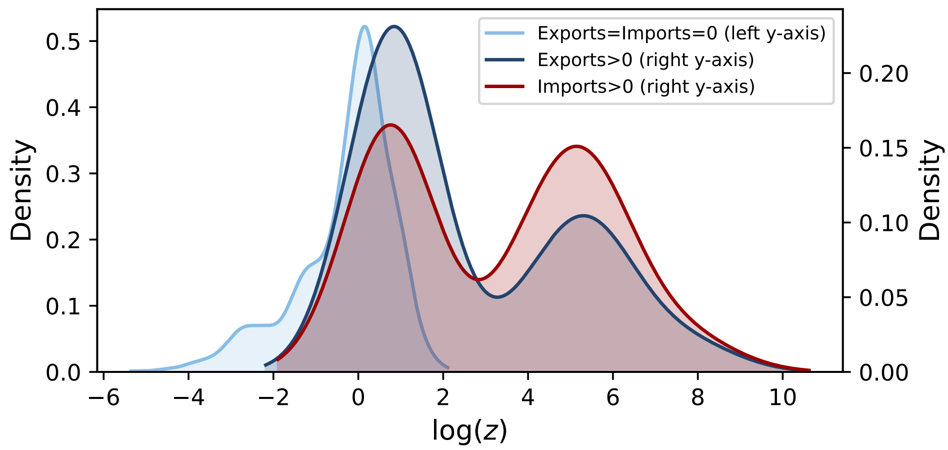


Figure 6: Distribution of Log Firm Size $\log(z)$ by Type

capture firm-level responses conditional on sectoral and aggregate variables.[15] We provide further details in Appendix C. The heterogeneous responses to the shocks are in turn used as unmatched moments to evaluate the model's ability to replicate the heterogeneity in output response across the firm-size distribution.

4.4.1 Model Fit to Empirical Impulse-Responses

Our estimated parameter values are displayed in Table 4, while Figure 7 displays our estimated IRFs to a foreign adverse supply shock as well as the model fit in partial equilibrium. The model fits the estimated output response very accurately. The dynamics of the materials expenditures are matched closely, with the labor response being at the high end compared to the empirical estimates, but still within the confidence bands. This response is needed to match the output response observed in the data, considering that our model lacks both capital and utilization. The estimated elasticity of substitution between labor and materials is restricted by the bounds we set in the estimation. The model has issues replicating the point estimate of the large price response found in the data, but the model response is within the confidence interval. We estimate a level of the elasticity of substitution of $\epsilon^P = 9.6$, which is in line with standard values used in the New-Keynesian literature. The value implies that the absolute response of real output is a magnitude larger the price response.

In Figure 8, we display the sectoral output and price responses in partial equilibrium, corresponding to the shock in Figure 7. In general, the largest responses are in the manufacturing sectors (categorized with a C). However, we do find a large response in

15. In the empirical analysis, we estimated the effect for the average firm in terms of the size distribution. In the matching exercise, we instead match the average response in the model to the average response (thus not for the firm of average size but the average response across all firms). Even so, the estimated impulse responses are very closely related to the estimates obtained for the average firm in terms of sales.

Table 4: Heterogeneous Firm Model Estimation

Parameter	Meaning	Value
ϕ^L	Adjustment cost - labor	10.1 (1.98)
ϕ^M	Adjustment cost - materials	0.004 (0.185)
ϕ	EoS between ℓ, m	1.0 (0.37)
ϵ^P	EoS between varieties	9.6 (2.27)

Notes: This table summarizes the estimated model parameters. Standard errors obtained using the Delta method are displayed in parenthesis.

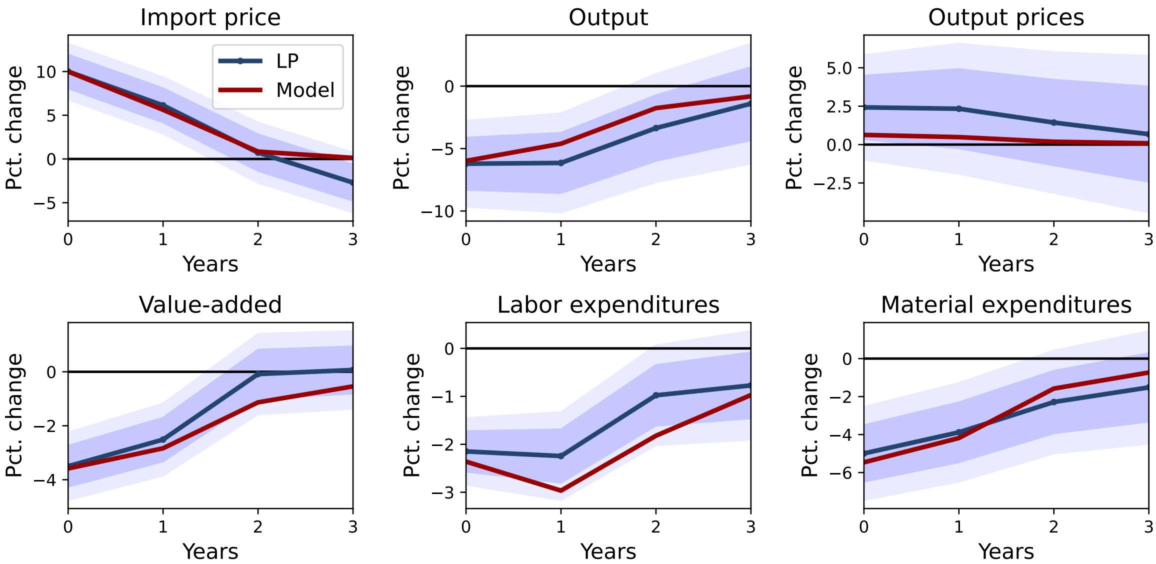


Figure 7: Impulse Responses to a Supply Shock vs. Model Fit

Notes: The Figure shows the dynamic impulse responses on several firm-level outcomes from a foreign supply shock, scaled to deliver a 10% increase in the import price. Blue is the empirical estimates and red is the model counterpart. Value-added refers to real value-added and materials to nominal materials expenditure. The standard errors are clustered at the sector-time level. 66 and 90 percent confidence intervals are reported as the shaded grey areas.
Source: Firm-level data are obtained from the FirmStat, Regnskab, Foreign Trade Statistics Register, and VARS registers from Statistics Denmark.

several of the sectors that are typically counted as non-tradable sectors (Di Giovanni et al., 2024). In particular, we find a large response in construction (F), transportation and storage (H), and information and communication (J). Thus, counting these sectors as non-tradables would have considerably underestimated the response to the shock in partial equilibrium.

4.4.2 Heterogeneous Responses

The fit of the average response in our sample is decent, but what about the heterogeneous responses? Given the insights from Section 2, we know that heterogeneity in firm size is an essential determinant of the aggregate response, so matching heterogeneous responses is crucial. Figure 9 displays the responses for real output z across the percentiles of the firm size distribution. The left Figure displays the estimated re-

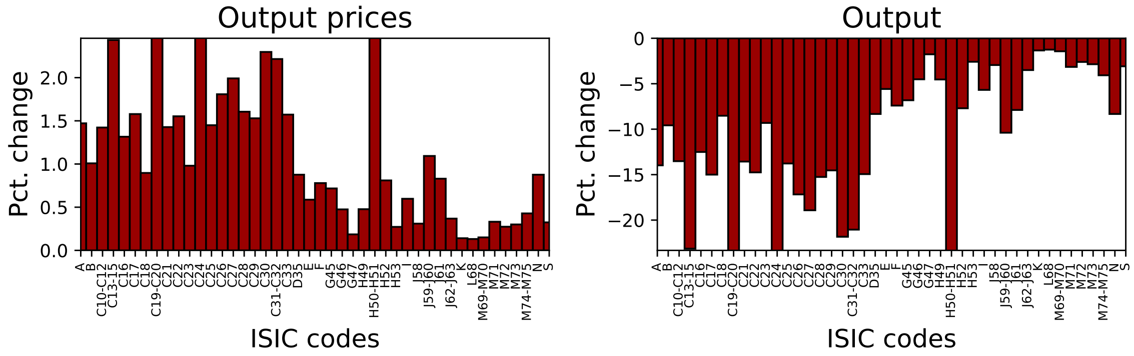


Figure 8: PE Responses by Sector

Note: Aggregate sector responses on impact following partial equilibrium import price shock for prices and real output.

sponses, while the right Figure shows the model counterpart. Considering that the responses across the size distribution are not a targeted moment in the estimation of the model, the fit is quite good. Specifically, we almost perfectly match the responses of the small and middle-size firms but underestimate the response of the firms at the largest percentiles.

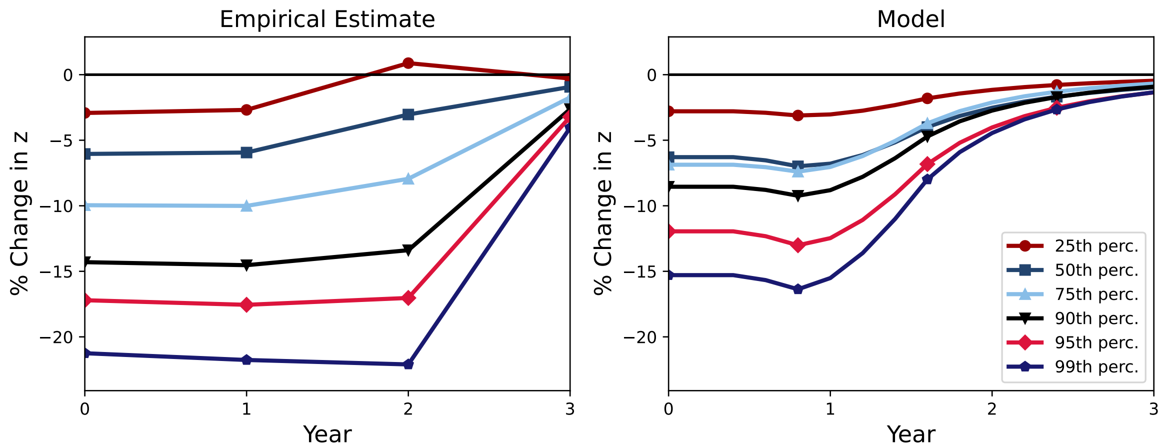


Figure 9: Heterogeneous Impulse Responses to a Supply Shock: Empirical vs. Model

Note: The left panel shows our empirical estimates of the real output response to a 10% shock to import prices, across the firm distribution as estimated from eq. (11). The right panel shows the model counterpart.

5 Aggregate Foreign Supply Shocks

In this section, we analyze the role of firm heterogeneity and adjustment costs using the model framework presented in the previous section. Consistent with the empirical estimations, we consider a stylized foreign supply shock, namely an exogenous increase in the import price of 3%. The shock is persistent and returns to the baseline level after two years (see Figure A.3).[16]

To illustrate the mechanisms in place, we start by investigating the partial equilibrium response to the shock. The partial equilibrium response corresponds to the shock considered in the propositions in Section 2, illustrating the amplifying role of heterogeneity in material shares combined with real rigidities. Next, we consider the full model taking all general equilibrium effects into account.

5.1 Micro Responses to a Supply Shock

We first consider the role of heterogeneity in the transmission of the import price shock in partial equilibrium. That is, we restrict our attention a setting in which all other aggregate, sectoral prices remain fixed and only consider a shock to the import price $P^{M,F}_{s,t}$ and see how this affects firm level outcomes. Figure 10 displays the impulse responses of total output Z to the shock in 3 models: 1) The baseline model with both sector and firm heterogeneity (Het. Firm), 2) a model with heterogeneous sectors, but a representative firm inside each sector (Rep. Firm), and 3) a variation of our heterogeneous firm model, where we include heterogeneity in firm size and one other source of heterogeneity (materials shares, import shares, or export shares), denoted "Het. Firm - Limited het.". All models include sector heterogeneity and are calibrated to match the same aggregates in the data.

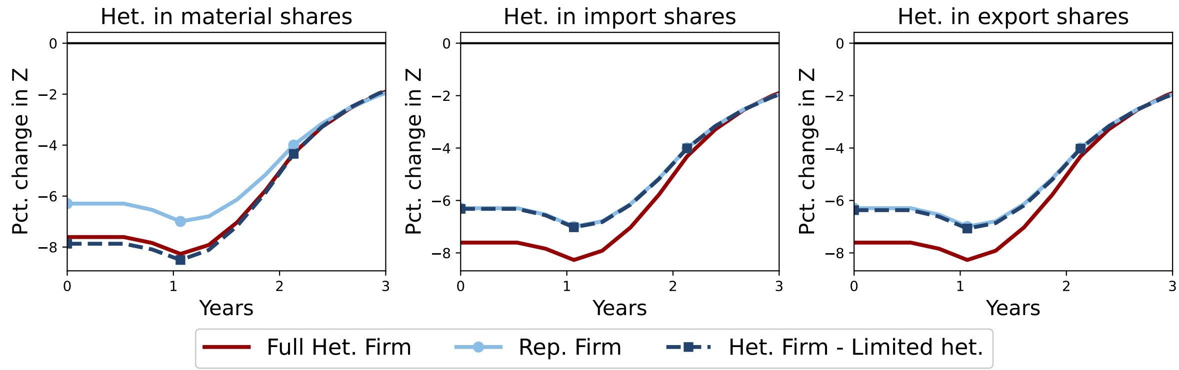
Figure 10 panel a) displays the aggregate output response in the three models. The increase in import prices leads to a drop in aggregate output of around 6% in the model with a representative firm within each sector. Adding firm heterogeneity amplifies the response by around 30%. Based on the theoretical insights from Section 2, this amplification is generated because the largest firms also rely on a larger share of flexible inputs because they are more materials-intensive. Also, heterogeneity in material shares combined with adjustment costs and non-unitary production function elasticities implies a further amplification as in Baqaee and Farhi (2019b). This reflects that with heterogeneity in material shares, firms are differently affected by the foreign shock which creates an amplification in itself. To establish these points, we consider a model with only heterogeneity in output and material shares. The response in output is almost identical to the response in the model with full heterogeneity, highlighting that the amplification is indeed a consequence of the positive connection between firm size and the share of flexible production inputs (materials). We also consider two other models with limited heterogeneity. Both have heterogeneity in size combined with heterogeneity in either

16. We obtain the shock profile based on an estimated SVAR model for 29 OECD countries, see Appendix A. For robustness, we also consider the impact of the full estimated shock, i.e., a shock taking changes in foreign demand and the foreign real interest rate into account. The overall conclusions carry over to this setting.

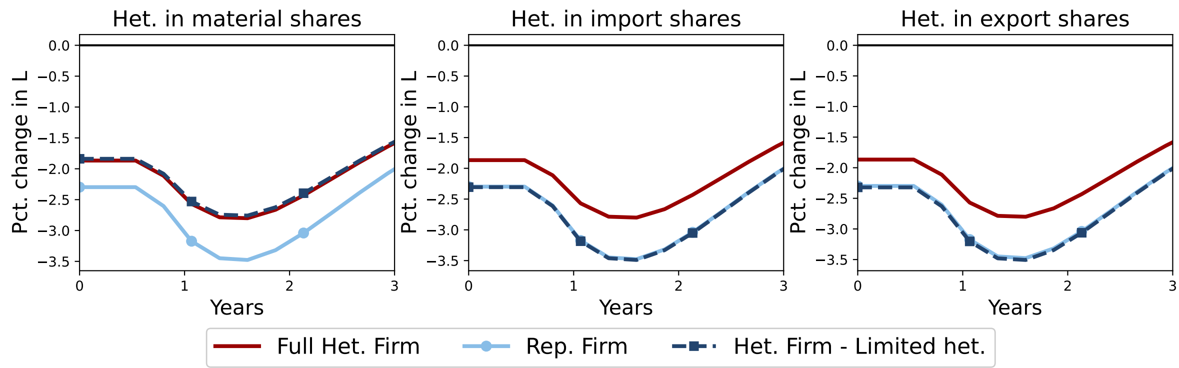
import or export shares. In both models, the output response is identical to the model with a representative firm within each sector. This confirms corollary 1, namely that heterogeneity in import shares and output alone is not enough to generate amplification. For the model with heterogeneity in export shares, the lack of amplification is because, in partial equilibrium, the firms are unaffected by the destination of sales because demand is held fixed.

In panel b) of Figure 10, we next consider the response of labor to the foreign supply shock. As shown in Section 2, we expect that firm heterogeneity dampens the labor response to foreign shocks. The responses in partial equilibrium confirm this dampening. The models with limited heterogeneity illustrate that, as for the output response, heterogeneity in materials shares is a necessary condition for the dampening effect.

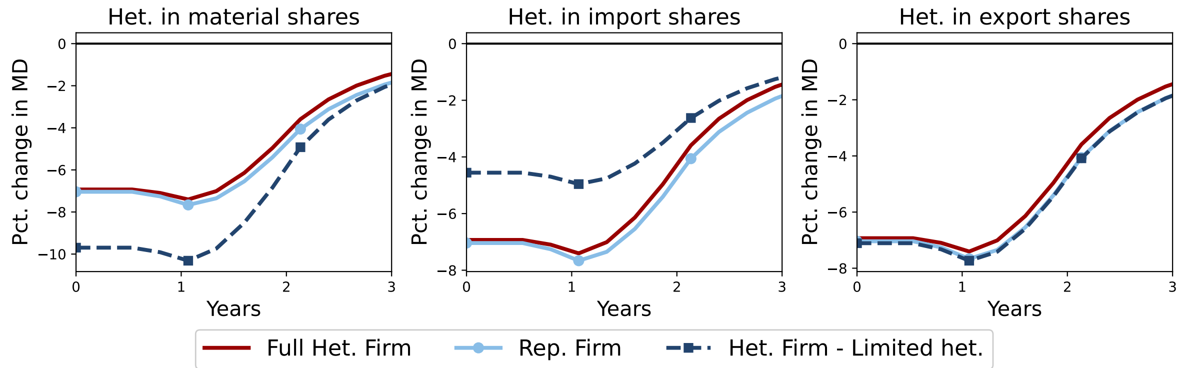
In our model, changes in GDP are either driven by labor or materials. In panel c) of Figure 10, we plot the response of domestic materials demand in the different models. We find no amplifying or dampening effect on domestic materials demand from firm heterogeneity. This means that firm heterogeneity does not influence the upstream propagation of the shock. However, this turns out to be a byproduct of the calibration more than a general rule. In the model with only heterogeneity in materials shares and output, we find an amplifying effect of firm heterogeneity. The line of reasoning from before carries over. When the largest firms are also most materials-intensive and thus rely on more flexible production inputs, the response of domestic materials gets amplified. However, it turns out that this amplification in domestic materials is offset by heterogeneity in import shares. When the largest firms are import-intensive, they are less connected upstream to domestic firms. Therefore, import heterogeneity dampens the domestic materials' response to foreign shocks.



(a) Response of Output



(b) Response of Labor



(c) Response of Domestic Materials

Figure 10: Supply Shock Responses With Different Levels of Heterogeneity

Proposition 2 shows that sectors with a high within-sector variance of materials and covariance between materials and output generate an amplification in the output response. In Figure 11, we plot the difference in responses between the baseline model with firm heterogeneity and the model with a representative firm, $\left|\frac{dX_t^{Het.firm}}{X_t^{Het.firm}}\right| - \left|\frac{dX_t^{Rep.firm}}{X_t^{Rep.firm}}\right|$ for $X = \{Z, L\}$. We plot these differences against the within-sector variance of materials and covariance between material intensity and output in steady state, $\mathbb{V}\text{ar}_i\left(\alpha_{i,s}\right)$ and $\mathbb{C}\text{ov}_i\left(\log z_{i,s}, \alpha_{i,s}\right)$. The plot confirms the positive connection between the covariance and shock amplification. The opposite holds true for labor, where the labor response in sectors with a high within-sector covariance is dampened. The same conclusions hold true for the variance of materials shares illustrating that sectors with large heterogeneity in materials shares experience an amplification of the output response, but a dampening of labor response.

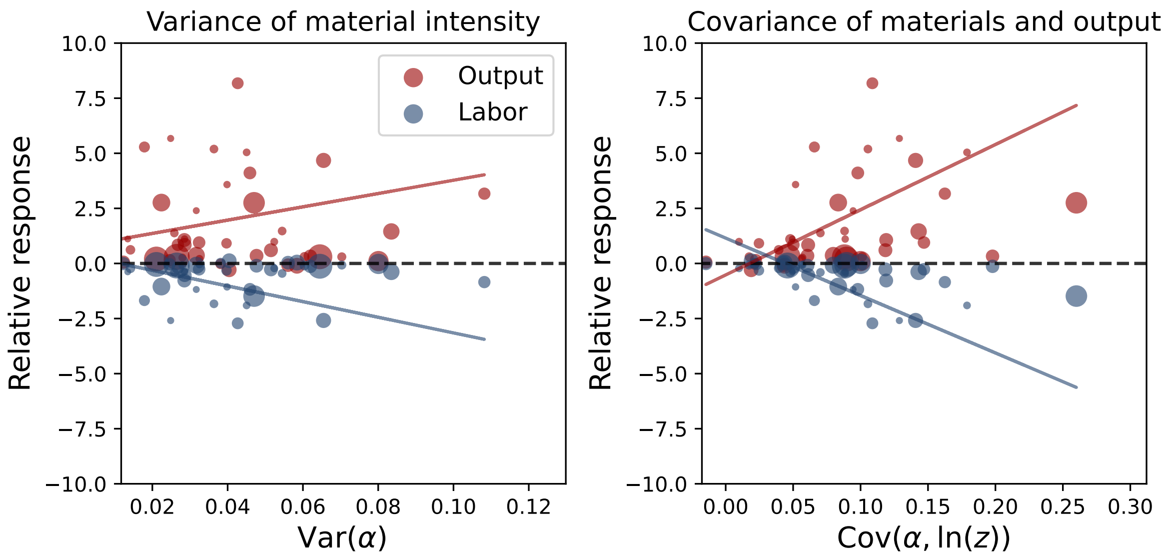


Figure 11: Amplification and Firm Heterogeneity

Note: The figure shows the difference in partial equilibrium impulse responses for sectoral output and labor between our baseline model and the model with only sectoral heterogeneity as a function of the within-sector variance of material shares and covariance with output.

Figure 12 displays the first-period response of output and labor with varying degrees of adjustment costs of labor. Without adjustment costs, the output response in the model with firm heterogeneity is identical to the response in the model with a representative firm within each sector. As the adjustment cost increases, it becomes more rigid to adjust the production inputs, which harms the firms with low materials share the most, implying that the response of output gets amplified. Oppositely, with low adjustment costs, the response of labor is dampened by firm heterogeneity. This is because the largest firms are less connected to the domestic labor market as they mainly apply materials in production. As the adjustment costs increase, the response in the two models move closer as with sufficiently high adjustment costs no change in labor is observed.

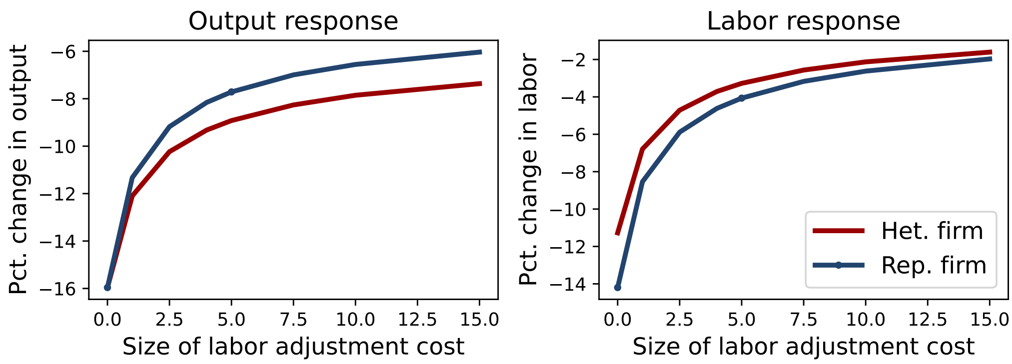


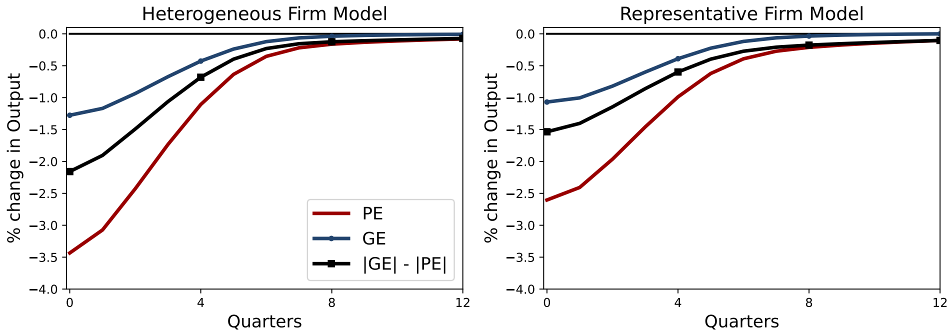
Figure 12: PE Output and Labor Responses for Varying Labor Adjustment Cost

Note: The figure displays the output and labor responses in the first period after the supply shock in the heterogeneous and representative firm model with a varying degree of the labor adjustment cost.

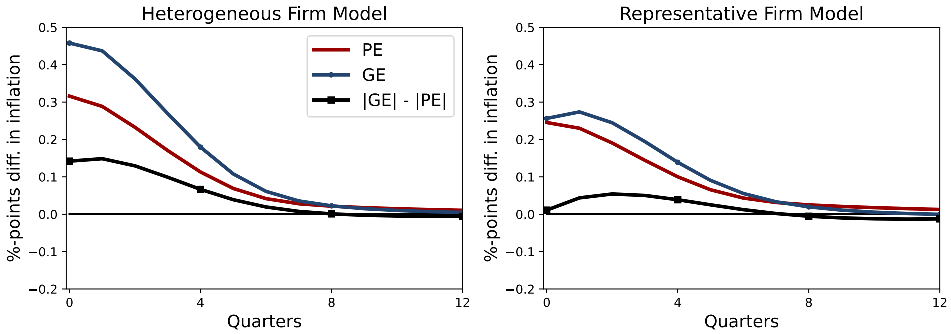
To summarize, the partial equilibrium analysis illustrates that firm heterogeneity in materials shares, combined with adjustment costs in labor, amplifies the output response. This amplification happens for two reasons. First, the heterogeneity of material shares amplifies the output response because firms are differently exposed to the foreign shock, leading to heterogeneity in the shocks themselves. Second, the fact that the largest firms rely on more flexible production inputs (i.e., a positive covariance between output and material shares) creates a further amplification. Whereas the amplification of the output response is increasing in the size of the labor adjustment cost, the dampening in the labor response is decreasing. Our analysis also shows that the output and labor response are invariant to heterogeneity in the import and export shares. It is important to note that the conclusions reached so far is in a partial equilibrium setting. In the next section, we consider the general equilibrium response to the shocks.

5.2 General Equilibrium Responses to a Supply Shock

We now consider how the supply shock affects the economy in general equilibrium. In moving from partial to general equilibrium, we find a substantially lower response of output in our full baseline model as well as in the standard model (Figure 13). The dampening is substantial, with the GE impulse being roughly half of the PE impulse. The picture is the opposite of the response of producer prices. Here, we find a substantially higher GE response. As we show below, this is primarily driven by a competition effect and increases in domestic materials prices. Notably, the amplification of the price response when moving from PE to GE is significantly larger in our heterogeneous firm model compared to the representative firm model.



(a) Response of Output



(b) Response of Producer Prices

Figure 13: Partial vs. General Equilibrium Responses

Note: Responses for output and inflation in the baseline model to a temporary 3% increase in import prices in the partial and general equilibrium model. The shock uses the estimated profile from the SVAR in Figure A.3.

5.2.1 Transmission in GE Model

Figure 14 displays the response of domestic key variables to the shock. The foreign shock generates what resembles a supply shock in the domestic economy: There is a persistent decline in GDP and employment lasting for around 2 years, with a simultaneous increase in domestic CPI. The real interest rate increases initially do declining expected inflation following the initial burst, which causes a decline in domestic consumption in the baseline model due to intertemporal substitution. The presence of nominal wage rigidities implies that real wages decline in the face of higher prices. This causes firms to substitute toward labor, thus alleviating some of the decline caused by lower demand. Comparing our heterogeneous firm model with the standard representative firm model we see that our earlier results from the partial equilibrium setting carries through to general equilibrium: The heterogeneous firm model predicts a lower response of labor and GDP, but generates amplification in output and prices. The amplification is notably very large for the response of the domestic CPI, which carries through to domestic consumption which also falls significantly more.

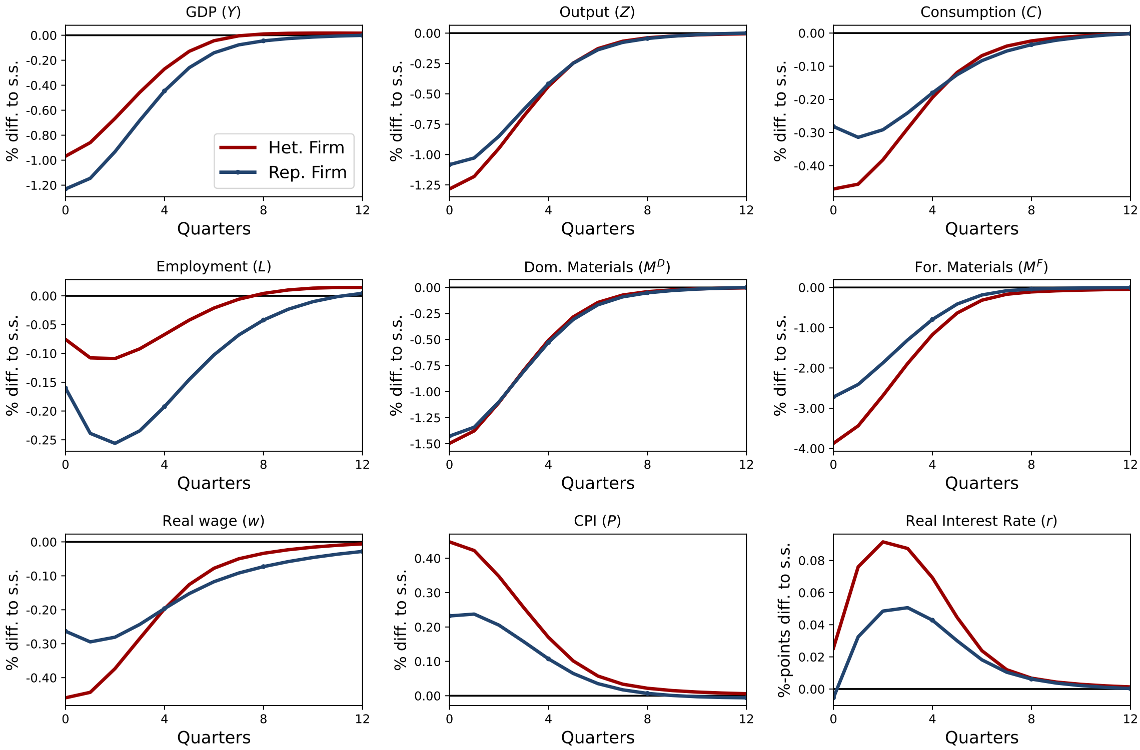


Figure 14: General Equilibrium Responses to Foreign Supply Shock With and Without Firm Heterogeneity

Note: Responses for several key variables in the heterogeneous firm model and the representative firm model to a temporary 3% increase in import prices. The shock uses the estimated profile from the SVAR in Figure A.3.

Figure 15 decomposes the response of GDP, output, and inflation in the heterogeneous firm model and shows that the GE dampening of the output response we obtain derives primarily from a competition effect. An increase in the competing price of goods within sectors has a stabilizing effect on output because firms can sell more goods overall given their price. Given that we estimate a sizeable elasticity of substitution within a sector ($\epsilon^P \approx 9$), this effect turns out to be quantitatively significant and reduces the drop in output by roughly two-thirds. The drop in demand induced by the supply shock in general equilibrium contributes to a slightly larger drop in output. Similar mechanisms carry over to the drop in GDP, except we find a sizable dampening role of the wage. This is because of a drop in the real wage, which creates a substitution effect towards labor. The increase in producer prices is driven by higher domestic and foreign prices of materials, which, through supply chains, also generates within-sector competition effects.

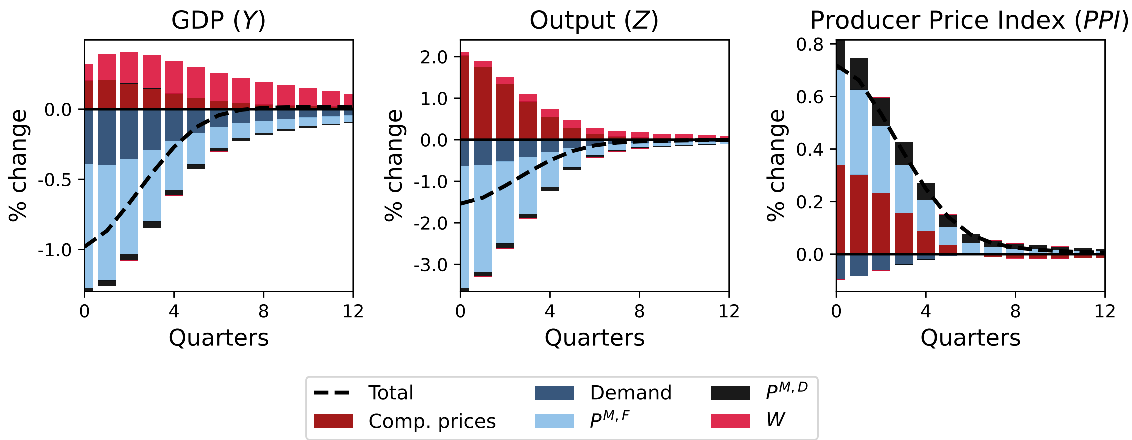


Figure 15: Decomposition of GDP, Output, and Inflation Responses

Note: This Figure decomposes the response of GDP, output, and inflation in the general equilibrium model into contributions from the various firm inputs. The Figure considers the responses to the stylized foreign supply shock, i.e., the import price shock. "Demand" refer to effects from $Z^D_{s,t}$, $Z^X_{s,t}$, "competitors price" refers effects from sectoral prices $P_{s,t}$.

5.2.2 Influence of Firm Heterogeneity

In partial equilibrium, firm heterogeneity amplifies the output response but dampens the labor response. In this section, we investigate if these results carry over to general equilibrium, where a multitude of new channels are active. In Figure 16, we display the difference in GDP, output, and producer price inflation responses between the heterogeneous model and the model with a representative firm within each sector. As in partial equilibrium, firm heterogeneity amplifies the output response. A decomposition of the %-point difference in output response reveals that the main source behind this amplification in GE is the same as in PE, namely the positive covariance between firm size and exposure to the shock. The larger output response in the heterogeneous firm model also implies a further amplification from a drop in demand. However, the amplification from firm heterogeneity is partly offset by the competitors' prices. When the largest firms are also most exposed to the shock because they import more, this implies an amplification of the producer price inflation. Thus, the competitors increase their prices more in the model with firm heterogeneity, allowing the firms to sell more without experiencing the same drop in demand.

In the absence of capital or any factor utilization, the GDP response is closely related to the labor response. Therefore, the same intuition from the labor responses carries over to the GDP response. Firm heterogeneity dampens the GDP response to the foreign shock by around 0.25%-point. This dampening is mainly driven by the influence of the import price, i.e., the shock we considered in partial equilibrium. The same logic as in the partial equilibrium applies: When larger and less labor-intensive firms are most exposed to the foreign shock, this creates a dampening in the GDP response.

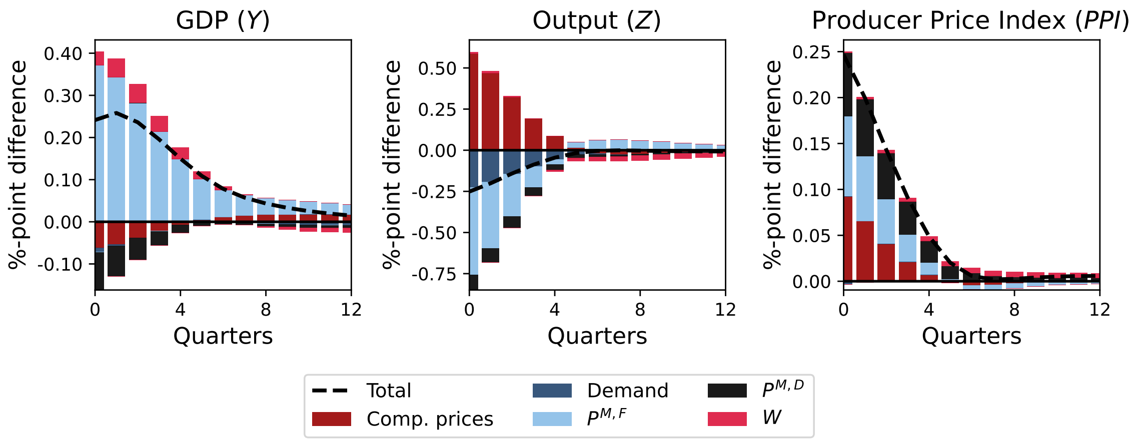


Figure 16: Sources of Amplification From Firm Heterogeneity

Note: This Figure decomposes the difference in response between the baseline model and the model with only sectoral heterogeneity into contributions from the various firm inputs. The Figure considers the responses to the stylized foreign supply shock, i.e., the import price shock. "Demand" refer to effects from $Z^D_{s,t}$, $Z^X_{s,t}$, "competitors price" refers effects from sectoral prices $P_{s,t}$.

In Figure A.16 in Appendix F, we plot the on-impact response in GE of output and labor for varying degree of adjustment costs. The figure display the same picture as in Figure 12: Increasing adjustment costs implies an amplification of the output response, but decreasing dampening in the labor response.

5.2.3 Robustness

We consider several robustness checks for our general equilibrium impulse responses - see Appendix F. These include 1) Flexible wages, 2) A floating exchange rate, 3) A fixed share of hand-to-mouth households in the population, and 4) different elasticities of substitution. Though these different model mechanisms do affect the impulse responses, the differences between our two model specifications remain more or less the same.

We also consider a supply shock giving rise to not only changes in import prices but also in foreign demand C^*_t and interest rates i^*_t. We estimate this shock using a sign-restricted SVAR, see Appendix E. The main conclusions about heterogeneity still stand. A decomposition of the response to shocks illustrates that the most important source of shock transmission is the change in the import price, i.e., the stylized supply shock.

6 Conclusion

This paper contributes to the understanding of supply chain disruptions in the context of foreign supply shocks, emphasizing the crucial role of firm heterogeneity and adjustment costs. We start with two theoretical insights. First, even though firm heterogeneity implies that firms are differently exposed to foreign shocks, this is not sufficient to generate an amplification of the inflation and output response. However, when combined with adjustment costs, the response of inflation and output are amplified relative to a model with a representative firm. This happens because larger firms mainly rely on flexible production inputs (materials) and can therefore easier adjust their production. Second, even in the absence of adjustment costs, firm heterogeneity generates a dampening in the labor response. This happens because the large firms are less connected to the domestic labor market because they mainly apply materials.

We provide empirical evidence of these predictions in a set of stylized facts. Large firms are systematically more material-, import-, and export-intensive and more responsive to foreign shock. We also provide a justification for an adjustment cost on labor by illustrating the significant difference between labor and materials in the response to foreign shocks.

We deploy a New-Keynesian multi-sector model with heterogeneous firms, calibrated to the universe of Danish firms and the empirical evidence, to illustrate that the theoretical insights carry over to a general equilibrium setting. In sum, our results provide evidence of the dual role of firm heterogeneity by illustrating the dampening effect on GDP but amplifying effect on inflation and output. This challenges conventional models that consider only heterogeneous sectors and a representative firm within each.

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A Appendix

A Empirical Appendix

A.1 Data

In this Appendix, we outline the construction of our data in detail. We apply two subsets of the same dataset, where the first is used for calibration of the quantitative model and the second for estimation of the direct effects.

Calibration sample To calibrate our model, we first require information on the sectors aggregate sales, materials expenditure, labor compensation, imports, and exports. This information is based on the Danish Input-Output tables from Statistics Denmark. We use the year 2005. We apply these aggregates to calibrate the share of each sector in the total flows of the Danish economy. We also apply these aggregates to measure the average productivity, labor share, import share, and export share within the sectors. By applying these aggregates, we ensure that our sample is representative of the Danish economy.

We can, in general, interpret these aggregates as a description of the between-sector heterogeneity. In Table A.1, we display the sector's share in aggregate sales and their sector shares. The sectors are based on the ISIC rev 4., but we choose to aggregate some sectors. This is because, in some cases, the sector has an average markup below one, which is not compatible with firms being substitutes. We also exclude all public sectors as our firm-level sample does not cover public firms. The resulting number of sectors is 44.

The heterogeneity within the sectors is calibrated based on our firm-level data. We draw information from four Danish registers from Statistics Denmark. Firms are identified across all registers by a single firm identifier (CVRNR). We restrict attention to 1999-2017 as the registers primarily include industrial firms before 1999.

The Danish Firm Statistics Register (FirmStat) and the accounting statistics dataset (Regnskab) cover the universe of Danish private-sector firms, except agriculture, financial institutions, and public administration. We obtain the CVRNR, sector code (six-digit NACE code), number of full-time employees, sales, labor compensation, materials, and value-added reported in Danish Kroner (DKK) from these datasets. We also obtain firm imports and exports from these registers. These flows are reported at the firm-year level. That is, aggregated across countries and products. Even so, the main advantage of using this dataset in our calibration, compared to customs data on trade at the product and country level, is that our sample includes service trade. This implies that we do not count sectors that are typically counted as service sectors as non-tradables. Instead, the degree of trade within a sector is entirely driven by the share of trading firms.

We impose a set of restrictions on the data. Our first restriction is to only use firms with positive sales, labor compensation, and materials expenditure. The last restriction on positive materials removes very few firms and is only imposed in the calibration sample. Next, we restrict attention to firms with at least five employees, as very small firms often have imputed data. In Table A.3 and A.3, we display some summary statistics about the coverage of the resulting sample.

The resulting dataset is used to calibrate the within-sector heterogeneity, namely the standard deviations and correlations between log sales, labor share, import share, and export share. We also calibrate our model for each sector to match the number of importing firms, exporting firms, and firms with both import and export. These correlations and shares of trading firms are reported in Table A.2. On average, sales correlate negatively with the labor share and positively with the import share and export share. This mimics the Danish firm-level data as closely as possible and replicates the patterns reported in Section 3. We also see that trade is far from limited to manufacturing firms. As an example, the category "Wholesale and retail trade" has a share of trading firms above 50 percent and constitutes around 27% of the number of firms in the sample. Thus, counting this sector as non-tradable severely underestimates the number of trading firms.

Table A.1: Sector Summary Statistics

Name	ISIC	Sector share	Labor share	Import share	Export share
Agriculture	A	0.027	0.151	0.196	0.199
Mining and quarrying	B	0.026	0.291	0.336	0.576
Food, bev., and tobacco	C10-C12	0.06	0.175	0.252	0.528
Textiles, apparel, and leather	C13-15	0.005	0.245	0.539	0.673
Wood, cork, except furniture	C16	0.007	0.308	0.403	0.279
Paper and paper products	C17	0.005	0.283	0.466	0.35
Media reproduction	C18	0.006	0.362	0.354	0.102
Petroleum, chem. products	C19-C20	0.022	0.12	0.411	0.513
Pharmaceutical products	C21	0.02	0.256	0.412	0.762
Rubber, plastic products	C22	0.01	0.32	0.51	0.451
Non-metal mineral products	C23	0.009	0.317	0.323	0.22
Basic metals	C24	0.004	0.208	0.529	0.507
Metal products	C25	0.018	0.351	0.396	0.24
Electronics, optics	C26	0.013	0.34	0.462	0.635
Electrical equipment	C27	0.007	0.274	0.438	0.464
Machinery and equipment	C28	0.039	0.283	0.403	0.588
Vehicle manufacture	C29	0.004	0.314	0.495	0.528
Other transport equipment	C30	0.004	0.188	0.472	0.479
Furniture and other mfg.	C31-C32	0.015	0.302	0.45	0.659
Machinery services	C33	0.004	0.333	0.434	0.113
Energy supply	D	0.022	0.148	0.155	0.236
Water, waste management	E	0.011	0.226	0.144	0.006
Construction	F	0.09	0.295	0.246	0.044
Auto trade, repair	G45	0.017	0.421	0.303	0.054
Wholesale trade	G46	0.082	0.42	0.222	0.381
Retail trade	G47	0.035	0.562	0.123	0.001
Land transport, pipelines	H49	0.03	0.365	0.199	0.185
Water, air transport	H50-H51	0.065	0.068	0.923	0.918
Transport support	H52	0.016	0.382	0.207	0.247
Postal, courier activities	H53	0.008	0.535	0.17	0.122
Accommodation, food	I	0.019	0.368	0.245	0.02
Publishing activities	J58	0.011	0.35	0.13	0.09
Media production	J59-J60	0.007	0.338	0.348	0.042
Telecommunications	J61	0.022	0.222	0.198	0.091
IT services	J62-J63	0.021	0.397	0.174	0.188
Financial services	K	0.052	0.481	0.077	0.028
Real estate	L	0.082	0.194	0.023	0
Legal, consulting activities	M69-M70	0.016	0.621	0.13	0.078
Engineering, analysis	M71	0.022	0.389	0.131	0.213
Scientific R and D	M72	0.005	0.586	0.215	0.171
Advertising, market research	M73	0.009	0.248	0.076	0.096
Professional, technical activities	M74-M75	0.005	0.447	0.173	0.052
Admin, support services	N	0.032	0.426	0.361	0.043
Other services	S	0.017	0.538	0.168	0.005

Notes: The table displays summary statistics of the sector-level variables. Sector share is the sector's share in total sales. The import share is defined as import relative to total firm materials, and the export share is defined relative to firm sales.
Source: Data is obtained from Statistics Denmark.

Table A.2: Firm Correlations

Name	ISIC	Firms	$\rho(labor, sales)$	$\rho(import, sales)$	$\rho(export, sales)$	Share imports	Share exports
Agriculture	A	66	-0.441	-0.119	0.463	0.278	0.434
Mining and quarrying	B	149	-0.065	0.18	0.446	0.559	0.565
Food, bev., and tobacco	C10-C12	2299	-0.704	0.248	0.481	0.381	0.408
Textiles, apparel, and leather	C13-15	623	-0.646	0.355	0.513	0.829	0.739
Wood, cork, except furniture	C16	690	-0.356	0.289	0.18	0.643	0.547
Paper and paper products	C17	218	-0.382	0.276	0.324	0.85	0.78
Media reproduction	C18	1071	-0.324	0.189	0.245	0.363	0.502
Petroleum, chem. products	C19-C20	275	-0.416	0.45	0.319	0.908	0.881
Pharmaceutical products	C21	94	-0.381	0.332	0.286	0.907	0.869
Rubber, plastic products	C22	701	-0.385	0.398	0.397	0.862	0.851
Non-metal mineral products	C23	474	-0.231	0.006	-0.013	0.701	0.541
Basic metals	C24	239	-0.535	0.535	0.504	0.748	0.772
Metal products	C25	3043	-0.419	0.293	0.298	0.535	0.556
Electronics, optics	C26	686	-0.354	0.328	0.305	0.899	0.856
Electrical equipment	C27	521	-0.419	0.377	0.271	0.84	0.812
Machinery and equipment	C28	1966	-0.409	0.314	0.416	0.788	0.815
Vehicle manufacture	C29	232	-0.313	0.439	0.553	0.791	0.701
Other transport equipment	C30	148	-0.395	0.384	0.301	0.835	0.851
Furniture and other mfg.	C31-C32	1231	-0.426	0.228	0.252	0.745	0.669
Machinery services	C33	1390	-0.268	0.192	0.163	0.535	0.57
Energy supply	D	144	-0.257	0.258	0.144	0.473	0.522
Water, waste management	E	243	-0.091	-0.005	0.062	0.525	0.596
Construction	F	18505	-0.292	-0.023	-0.077	0.136	0.071
Auto trade, repair	G45	4173	-0.64	0.167	0.023	0.376	0.333
Wholesale trade	G46	11571	-0.491	-0.018	0.086	0.837	0.695
Retail trade	G47	12125	-0.444	0.051	0.081	0.4	0.306
Land transport, pipelines	H49	4855	-0.196	0.082	-0.094	0.079	0.214
Water, air transport	H50-H51	463	-0.422	0.04	-0.167	0.322	0.667
Transport support	H52	1120	-0.388	0.055	0.07	0.414	0.598
Postal, courier activities	H53	351	-0.054	0.357	-0.064	0.151	0.127
Accommodation, food	I	7348	0.161	0.054	0.391	0.163	0.019
Publishing activities	J58	915	-0.083	-0.085	-0.017	0.394	0.461
Media production	J59-J60	400	-0.328	0.109	0.015	0.528	0.436
Telecommunications	J61	310	-0.307	0.222	-0.161	0.59	0.394
IT services	J62-J63	3702	-0.177	0.075	0.04	0.448	0.482
Financial services	K	486	-0.302	0.153	0.148	0.199	0.215
Real estate	L	2882	-0.348	0.046	-0.06	0.104	0.063
Legal, consulting activities	M69-M70	3701	-0.145	0.015	-0.063	0.121	0.228
Engineering, analysis	M71	2241	-0.251	0.064	0.126	0.245	0.297
Scientific R and D	M72	392	-0.24	0.222	0.245	0.675	0.522
Advertising, market research	M73	1543	-0.375	0.017	0.048	0.35	0.494
Professional, technical activities	M74-M75	1274	-0.047	0.229	0.147	0.331	0.336
Admin, support services	N	6575	-0.41	-0.043	0.026	0.173	0.143
Other services	S	790	-0.32	0.191	0.233	0.436	0.399

Notes: The table contains information on the number of firms, correlations within a sector, and share of importing firms and exporting firms. labor refers to the labor share in total expenses, import to the import share in materials, export to the export share in sales, and sales refers to the log of firm sales.
Source: Firm-level data are obtained from the FirmStat, Regnskab, and Foreign Trade Statistics Register.

Table A.3: Summary Statistics by Sample

	Firms	Average Sales	Average shares Labor	Import	Export	Share of sample Firms	Sales	Import	Export
All firms	287,974	16.657	0.472	0.048	0.056	1.000	1.000	1.000	1.000
Calibration data	96,900	45.629	0.486	0.065	0.079	0.336	0.922	0.946	0.966
Estimation data	24,179	131.929	0.346	0.185	0.176	0.084	0.665	0.701	0.552

Notes: The table displays summary statistics of firm variables. Results are shown both for the full sample of all firms with positive sales and labor compensation, the calibration sample with firms above five employees, and the estimation sample with firm-level price data. Sales are reported in Mio. DKK. The labor share is defined as the share of labor compensation in total firm expenditures (labor and material costs). The import share is defined as import relative to total firm materials, and the export share is defined relative to firm sales.
Source: Firm-level data are obtained from the FirmStat, Regnskab, Foreign Trade Statistics Register, and VARS registers from Statistics Denmark.

Table A.4: Share of National Accounts

	Sample share of national accounts Value-added	Export	Import
Calibration data	0.516	0.790	0.739
Estimation data	0.327	0.782	0.695

Notes: The table displays the main sample and the production data's coverage of national accounts. Value-added is defined as aggregate value-added in the sample relative to total private (non-public) value-added. Export and import are defined as the aggregate value in the sample relative to total material exports and imports in national accounts.
Source: Firm-level data are obtained from the FirmStat, Regnskab, Foreign Trade Statistics Register, and VARS registers from Statistics Denmark.

Estimation sample Our shift-share identification of the foreign supply shocks requires information on the firms' exports and imports at the product and country level. We obtain information on the firm's imports and exports at a detailed product- and destination-level from Danish customs data. The dataset contains trade flows at the 8-digit Combined Nomenclature, but we aggregate up to the HS6-level to be comparable with the Baci data from CEPII used to construct the instrument (Gaulier and Zignago, 2010). The flows are reported in values (DKK) and weight in kilos. We apply both to construct the unit trade values.

The sample presented this far only contains nominal values. However, the response in real and nominal sales may differ for various reasons. In particular, if shocks are passed on to domestic buyers by increasing prices (Amiti et al., 2019). Therefore, to properly match our quantitative model, we need firm-level price data.

To obtain real sales measures and investigate the pass-through of cost shocks to prices, we combine the FirmStat and the Regnskab datasets with the Manufacturers' Sales of Goods database (VARS), the Danish version of the Prodcom statistics regulated by Eurostat. The statistic is a quarterly survey of all firms in the manufacturing sector with at least ten employees. We restrict attention to firm-product flows that exist the entire year, which eliminates around 8% of the observations. The register contains the sales in value and volume at a detailed product level, enabling us to construct firm-specific price levels. We create these price levels by combining the VARS data with the unit values of Danish exports from the trade register.

We define the price level of firm f as a weighted average of the firm-specific prices across different destinations, c, and product categories, p:

$$P_{f,t} = \sum_{c,p} \omega_{f,c,p,t} P_{f,c,p,t}$$

where $\omega_{f,c,p,t}$ is the product p destination c share in total firm sales at time t.[17]

Summary statistics of the resulting estimation sample are displayed in Table A.3. Overall, the average firm in the estimation sample is larger than the average firm in the calibration sample and has larger import and export shares. This reflects that the majority of the firms in the estimation sample are trading. This is exactly the firms that are affected by the direct foreign supply shock, implying that even though relatively few firms are in the estimation sample, it still contains the majority of trading firms.

A.2 Shift-Share Design

As in any study using shift-share instruments, the exogeneity of the shocks should either originate from the shares or the shifters. Following Adao et al., 2019, our setting relies on exogeneity of the shifters. The logic behind this choice is that firms choose endogenously which market to source from and take the export supply of that market as given. Under this interpretation, the critical assumption is that the foreign supply of products, $S^{EX}_{c,p,t}$, is exogenous to shocks to individual Danish firms, corresponding to the identifying assumptions for shift-share instruments in Borusyak et al. (2022). Note that under this interpretation of the instrument, the identification is robust to endogenous movements in the import shares. Nevertheless, as firms may endogenously choose which market to source their goods from based on their information set at time t, we choose to lag the shares one period. In that case, the shares are only endogenous to the extent that information about the shifters at time t is already in the information set of the Danish firms at time $t-1$. To test this formally, we regress the shares, $s^{IM}_{i,c,p,t-1}$, on the growth rate in foreign export supply. If a positive connection exists, it implies that Danish firms trade toward markets that experience an increase in export supply, potentially driven by factors such as productivity shocks. We find no significant influence of the shares on the shifters, implying that the lagged shares appear exogenous.

Since we rely on exogeneity of the shifters for identification, we need a sufficient

17. We refer to Smeets and Warzynski, 2013 for a similar application on Danish data.

amount of variation in the source of the shocks to obtain consistency (Goldsmith-Pinkham et al., 2020). Borusyak et al. (2022) outline two criteria for consistency of the shift-share instruments. First, the shocks should be as good as randomly assigned, i.e., the shocks should be uncorrelated with other relevant unobservables. In our setting, this amounts to the shifters being exogenous to the individual firm. Second, the instruments should incorporate many sufficiently independent shocks, each with a relatively small exposure. Taking the year 2005 as an example, 419,715 unique potential markets in our sample exist, that is, unique combinations of countries and HS6 codes. Of the 9,292 Danish importing firms in our sample in that year, they are active in a total of 48,070 markets. Not only does this ensure a large sample of shocks - it is also unlikely that these are all correlated. In addition, it is also unlikely that a single market dominates: Each market only serves, on average, five different Danish firms (the median is two). Instrument relevance further holds if individual firms are only exposed to a small number of shocks. The median number of markets that a firm imports from is eight, highlighting that the individual firm is only exposed to relatively few shocks.

One of the advantages of using shift-share instruments is that we obtain instruments that are unique to each firm. We obtain heterogeneity from two sources. The first is the firm's decision on where to buy its goods from (i.e., the extensive margin of trade). Of the 9,292 firms, 8,786 operate on a unique combination of markets. Thus, only around 5% of firms operate on a combination of markets, which is identical to other Danish firms. The second source comes from the intensive margin, namely how much to import from each market. To evaluate the importance of these two margins, we regress the shocks on a set of market-fixed effects. The R-squared of this equation determines how large a share of the heterogeneity in the shocks that is generated on the extensive margin compared to the intensive margin. 67% of the variation in the instruments is generated by the extensive margin (the market effects) and the remainder by the intensive margin (the shares). Thus, both margins of trade contribute significantly to the heterogeneity of the shocks.

A.3 Empirical Results, Robustness

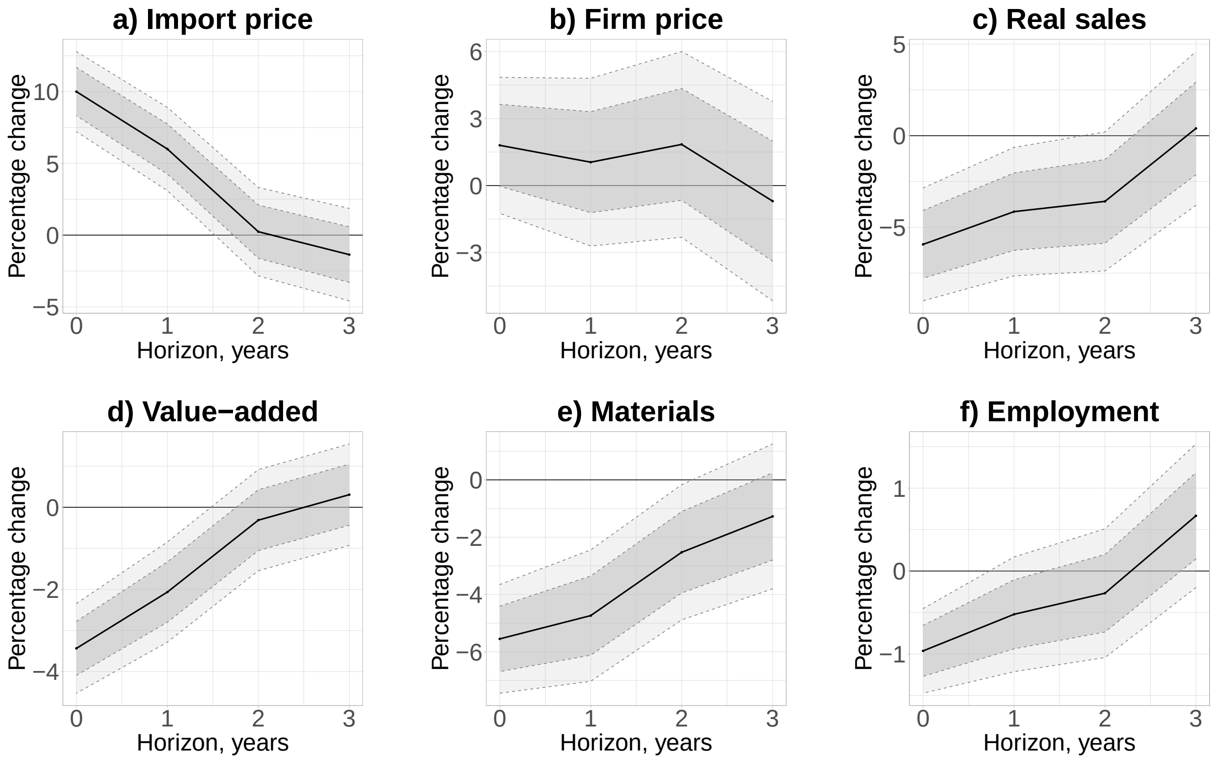

Figure A.1: Impulse-Responses to Negative Foreign Supply Shock, One Lag Included in Estimation

Notes: The Figure shows the dynamic impulse responses on several firm-level outcomes from a foreign supply shock, scaled to deliver a 10% increase in the import price. One lag is included of the dependent variable and the shocks. Value-added refers to nominal value-added and materials to nominal materials expenditure. The standard errors are clustered at the sector-time level. 66 and 90 percent confidence intervals are reported as the shaded grey areas.
Source: Firm-level data are obtained from the FirmStat, Regnskab, Foreign Trade Statistics Register, and the VARS dataset.

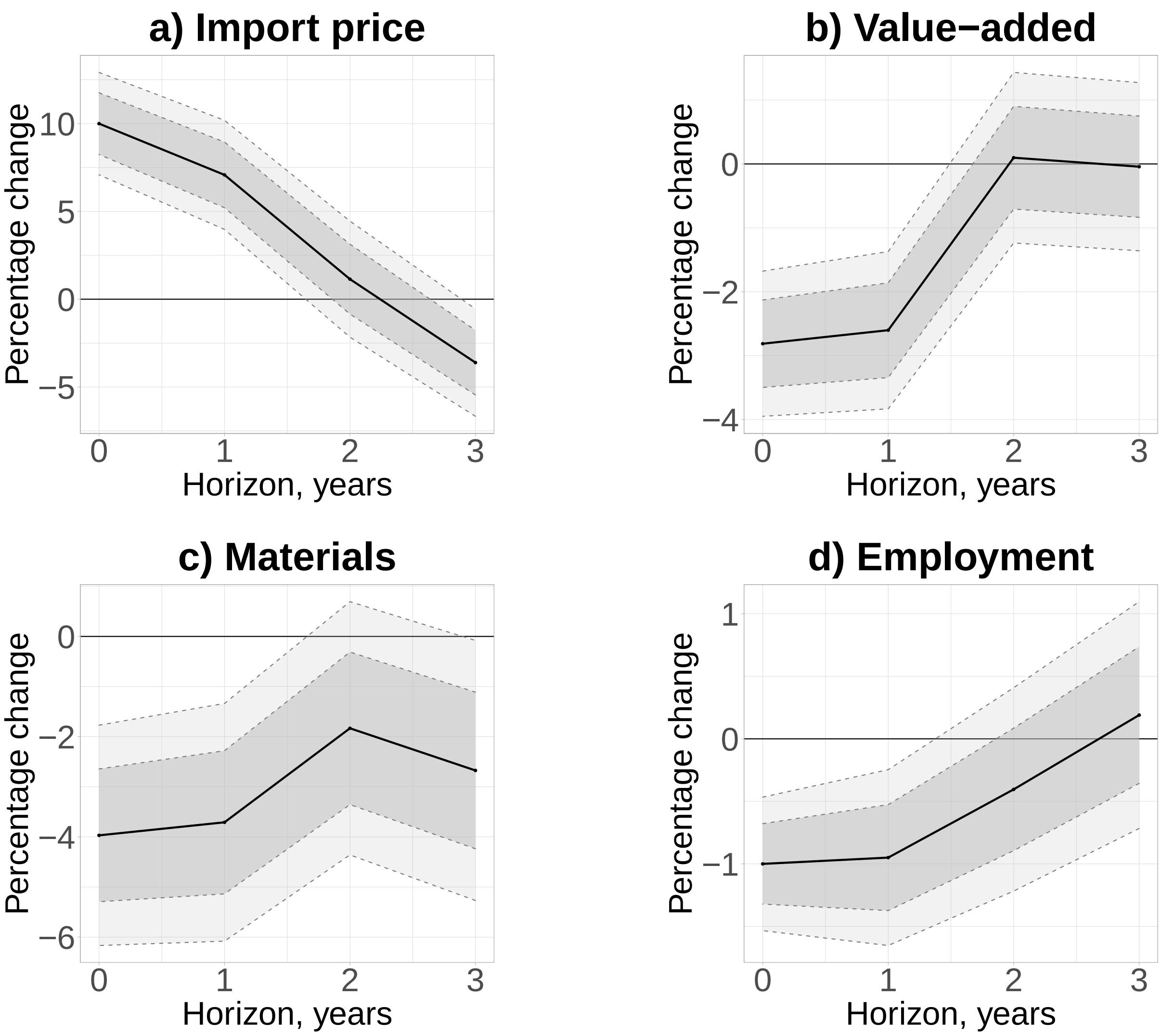

Figure A.2: Impulse-Responses to Negative Foreign Supply Shock, Calibration Sample

Notes: The Figure shows the dynamic impulse responses on several firm-level outcomes from a foreign supply shock, scaled to deliver a 10% increase in the import price. The calibration sample is used in estimation. The standard errors are clustered at the sector-time level. 66 and 90 percent confidence intervals are reported as the shaded grey areas.
Source: Firm-level data are obtained from the FirmStat, Regnskab, and Foreign Trade Statistics Register.

A.4 Structural VAR Estimations

Model specification Given the complexity of our small open economy model, we aim to keep the modeling of the foreign economy as simple as possible while still maintaining relevant empirical properties. We follow Christiano et al. (2011) and model the foreign economy as a five-variable VAR model:

$$\boldsymbol{X}_t = \sum_{p=1}^{P} \boldsymbol{\Phi}_{t-p} \boldsymbol{X}_{t-p} + \epsilon_t \tag{41}$$

where $\boldsymbol{X}_t = \left[\log GDP_t^*, i_t^*, \log P_{CPI,t}^*, \log P_{F,t}^*, \log C_t^*\right]'$ contains (appropriately de-trended) foreign GDP, policy rate, CPI, export price, and foreign imports. We uncover the underlying structural supply shock in (41) using sign restrictions. We impose:

$$\begin{pmatrix} - & ? & ? & ? & ? \\ ? & ? & ? & ? & ? \\ + & ? & ? & ? & ? \\ + & ? & ? & ? & ? \\ - & ? & ? & ? & ? \end{pmatrix}$$

Note that the only shock we identify is a foreign supply shock. Therefore, the estimated shock should be viewed as a partially identified shock. This shock is a negative foreign supply shock that implies a drop in GDP and foreign imports, but increase in the foreign CPI and export price.

We construct the foreign variables as a weighted average of 29 OECD countries.[18] The sample is unbalanced and covers the time period 1983Q1 to 2019Q4. We obtain the variables from the OECD Statistics database.[19] The weights are obtained as the 1995 share of Danish goods trade (imports and exports). That is, important trading partners of Denmark also receive a larger weight. We use two lags in the estimation and impose the sign restrictions for the first two quarters. We apply 5,000 accepted draws using the Uhlig (2005) rejection method.

Main estimation results In Figure A.3, we display the response of the foreign economy to a supply shock estimated based on the methodology outlined in Section 4. Foreign import demand drops on impact with around 1% and remains negative for around 10 quarters. Foreign CPI and export price increase initially by 0.2% and 0.7% and remain significantly above the steady state level for at least a year. At the same time, the real interest rate drops by 0.2%-point and remains below the initial level for around four years. However, this shock is estimated with large uncertainty.

Taken together, the identified shock resembles a negative foreign supply shock with a drop in foreign supply and demand but increases in prices. We next move on to investigate how this shock influences the Danish economy.

18. These countries are displayed in Appendix A.
19. All variables are seasonally adjusted. GDP, imports, CPI, and the export price is detrended using a HP-filter. The Policy rate is constructed as the real interest rate, where the interest rate is the short-term interest rate and inflation as the one-quarter CPI inflation. We detrend the policy rate with a constant and a linear trend.

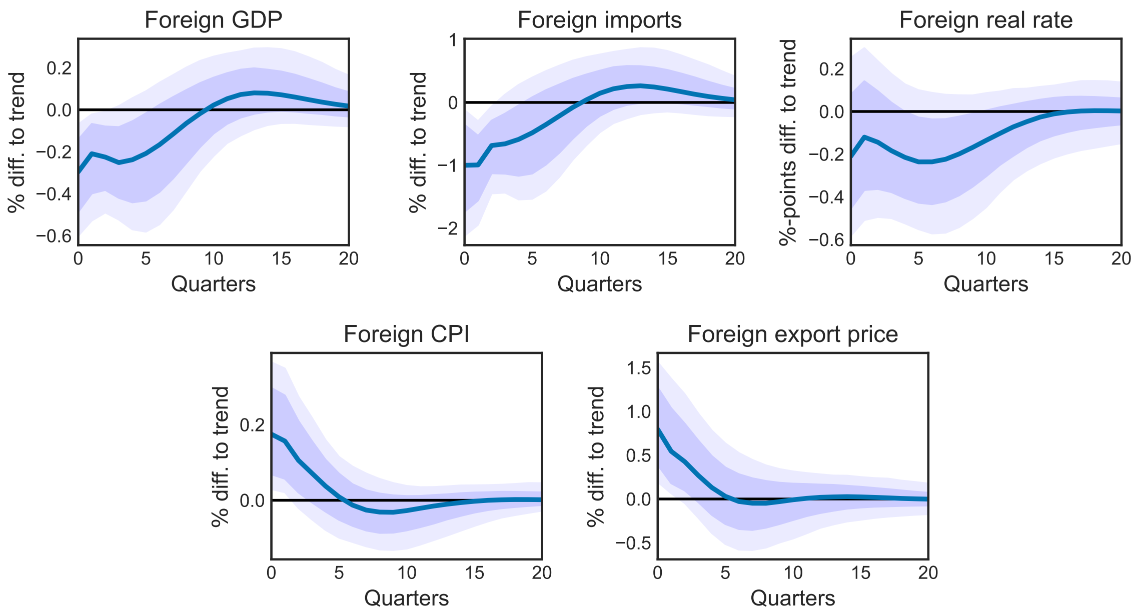

Figure A.3: Macro-Level Foreign Supply Shock

Note: The Figure shows the *foreign* responses to a foreign supply shock in the estimated VAR based on sign restrictions. The shaded areas represent the 68% and 90% confidence intervals.

Robustness Below we apply a wide range of robustness checks. First, we change the set of countries in the foreign economy. We consider two different sets of countries, the top 10 trading partners of Denmark and the G7 countries. The responses in the top 10 trading partners are almost identical to the baseline set of 29 OECD countries. The responses in the G7 countries are similar to the baseline, but is slightly more persistent and the response in the export price lower than the baseline.

Second, we detrend the data with a linear and quadratic trend. This implies slightly more persistent shocks than the baseline using the HP-filter, but the impact of the shock in the first year of the shock is very closely related to the shock with the HP filter.

Third, we apply a balanced sample by only using the time period 1995Q1 to 2019Q4. The responses are more noisy, which is a natural consequence of having fewer observations. Even so, the persistence and impact of the shocks remain rather unchanged.

Fourth, we apply 2015 trade weights instead of 1995. The responses are almost identical. However, it should be kept in mind that countries such as China are not in our sample. This is not an issue in 1995, where China is only the 15th most important trading partner (accounting for 1.9% of imports and 0.7% of exports), and the set of OECD countries in our sample account for 84% of Danish goods exports and 89% of imports. In 2015, China ranked as the 4th most important trading partner (accounting for 7.8% of imports and 4.8% of exports). Thus, leaving China out may have an impact on the representativeness of the set of countries. Even so, the countries in our sample still account for 78.4% of Danish imports and 79.4% of exports in 2015.

Fifth, we restrict the real interest rate to have a positive response following the shock. As the ECB only target inflation and the most important trading partners of Denmark are in the Euro area, we could expect the response to the supply shock as positive.[20] The resulting impulse-responses show that the real interest rate is above the steady-state level for at least a year and the other shocks also become marginally more persistent. Lastly, we apply four lags instead of the baseline number of 2. This leads to minor differences in the responses to the shock.

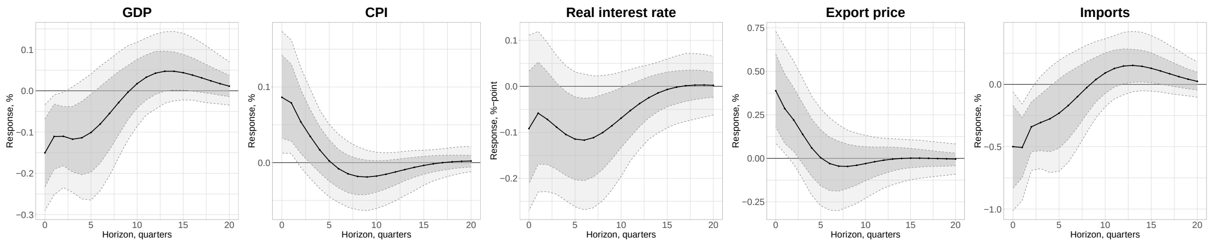

Figure A.4: Macro-Level Foreign Supply Shock - Top 10 Trading Partners

Note: The Figure shows the *foreign* responses to a foreign supply shock in the estimated VAR based on sign restrictions. The foreign economy is the top 10 trading partners in 1995. The shaded areas represent the 68% and 90% confidence intervals.

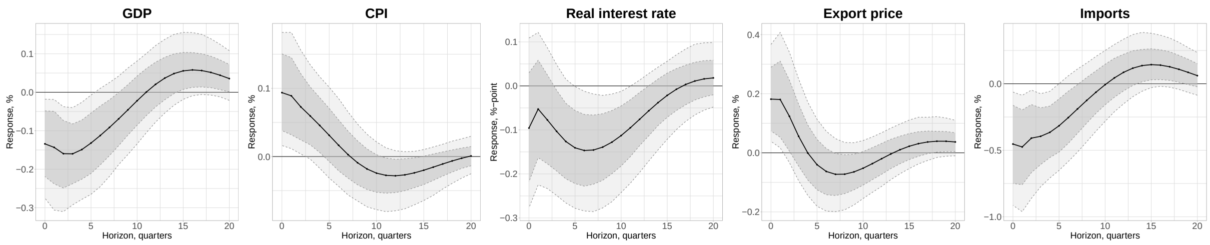

Figure A.5: Macro-Level Foreign Supply Shock - G7 Countries

Note: The Figure shows the *foreign* responses to a foreign supply shock in the estimated VAR based on sign restrictions. The foreign economy is the G7 countries weighted based on their share in DK trade in 1995. The shaded areas represent the 68% and 90% confidence intervals.

20. In an earlier version of the paper, we only applied the Euro-area as the foreign market and obtained a positive response in the interest rate as a response to the shock.

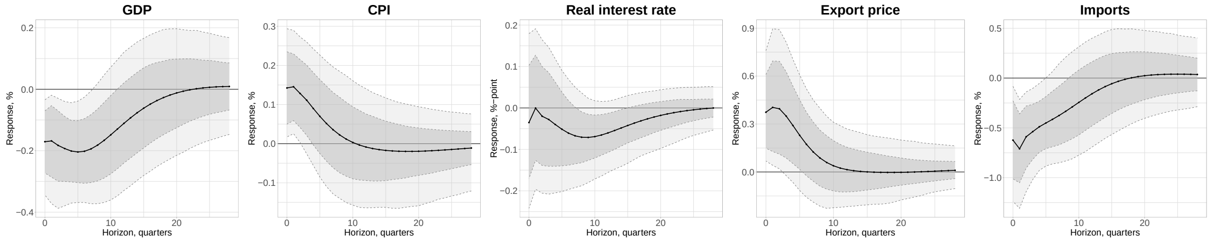

Figure A.6: Macro-Level Foreign Supply Shock - Quadratic Detrend

Note: The Figure shows the *foreign* responses to a foreign supply shock in the estimated VAR based on sign restrictions. The variables are detrended with a linear and quadratic trend. The shaded areas represent the 68% and 90% confidence intervals.

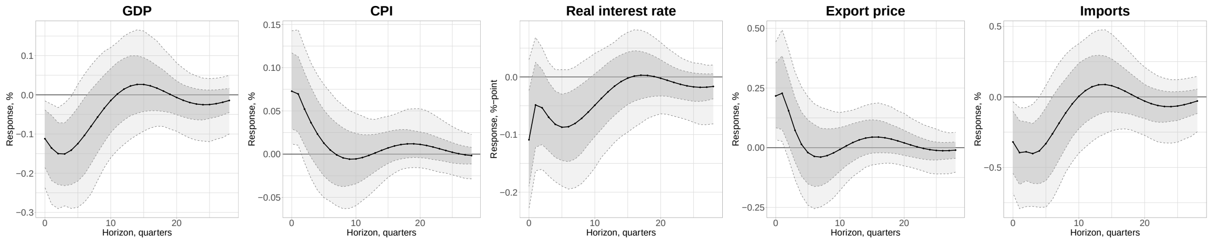

Figure A.7: Macro-Level Foreign Supply Shock - Balanced Sample

Note: The Figure shows the *foreign* responses to a foreign supply shock in the estimated VAR based on sign restrictions. The sample period is restricted to 1995Q1 to 2019Q4. The shaded areas represent the 68% and 90% confidence intervals.

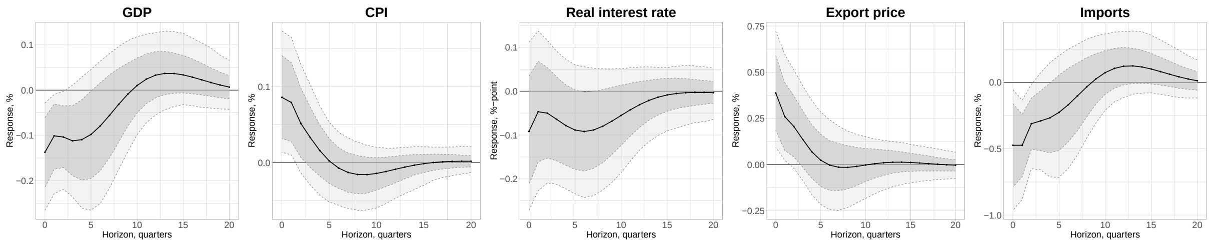

Figure A.8: Macro-Level Foreign Supply Shock - 2015 Trade Weights

Note: The Figure shows the *foreign* responses to a foreign supply shock in the estimated VAR based on sign restrictions. Trade weights are obtained from as the 2015 share in Danish goods trade. The shaded areas represent the 68% and 90% confidence intervals.

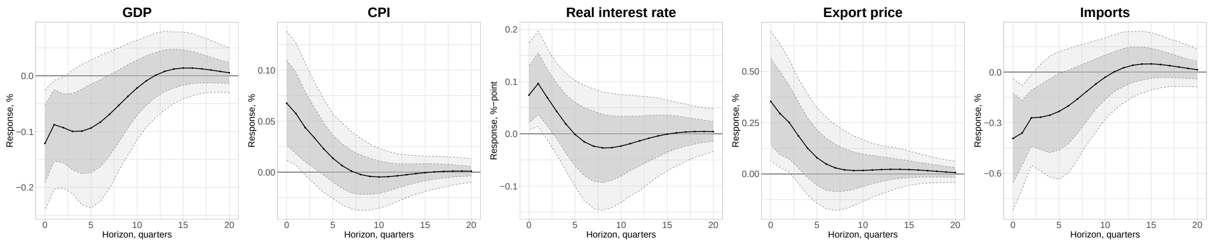

Figure A.9: Macro-Level Foreign Supply Shock - New Restrictions

Note: The Figure shows the *foreign* responses to a foreign supply shock in the estimated VAR based on sign restrictions. The real interest rate is restricted to a positive response in the first two quarters. The shaded areas represent the 68% and 90% confidence intervals.

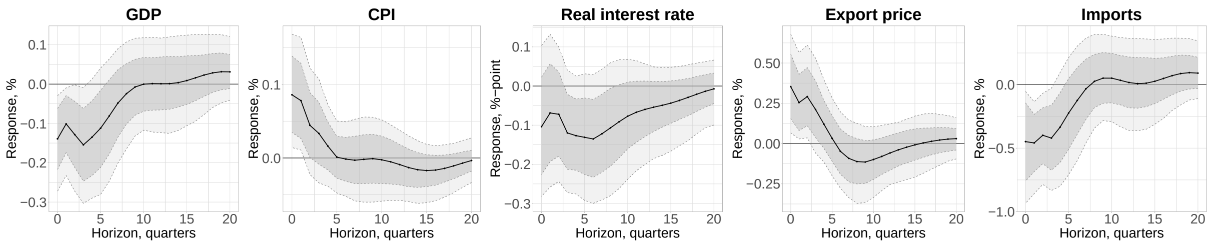

Figure A.10: Macro-Level Foreign Supply Shock - Four Lags

Note: The Figure shows the *foreign* responses to a foreign supply shock in the estimated VAR based on sign restrictions. Four lags included. The shaded areas represent the 68% and 90% confidence intervals.

B Model Appendix

B.1 Static Model Description and Solution.

The following equations make up the static model from section 2:

$$
\begin{aligned}
z_{i,s,t} &= \left[(1-\alpha_{i,s})^{\frac{1}{\phi}}\,\ell_{i,s,t}^{\frac{\phi-1}{\phi}} + \alpha_{i,s}^{\frac{1}{\phi}} m_{i,s,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \\
m_{i,s,t} &= \left[\gamma_{i,s}^{\frac{1}{\vartheta}}\left(m^{F}_{i,s,t}\right)^{\frac{\vartheta-1}{\vartheta}} + (1-\gamma_{i,s})^{\frac{1}{\vartheta}}\left(m^{D}_{i,s,t}\right)^{\frac{\vartheta-1}{\vartheta}}\right]^{\frac{\vartheta}{\vartheta-1}} \\
m^{D}_{i,s,t} &= \left[\sum_{j\in J}\Theta_{s,j}^{\frac{1}{\eta}} s_{j,i,s,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \\
z_{i,s,t} &= \varrho_{i,s,t}\left(\frac{p_{i,s,t}}{P_{s,t}}\right)^{-\epsilon^{P}} Z_{s,t} \\
\ell_{i,s,t} &= (1-\alpha_{i,s})\left(\frac{w_{s,t}}{mc_{i,s,t}}\right)^{-\phi} z_{i,s,t} \\
m_{i,s,t} &= \alpha_{i,s}\left(\frac{P^{M}_{i,s,t}}{mc_{i,s,t}}\right)^{-\phi} z_{i,s,t} \\
mc_{i,s,t} &= \frac{p_{i,s,t}}{P_{s,t}} - \lambda_{i,s,t} \\
\frac{p_{i,s,t}}{\epsilon^{P}} &= \lambda_{i,s,t}
\end{aligned}
$$

Because of constant returns to scale the firm model does not unique pin down the level of output in steady state. This allows to focus on a specific steady state where $z_{i,s,t} = z_{i,s}$ where $z_{i,s}$ is exogenous. Furthermore, with prices equal to 1 we get the following steady state:

$$
\begin{aligned}
mc_{i,s} &= \frac{1}{\mu} \\
\ell_{i,s} &= (1-\alpha_{i,s})\,(w_s\mu)^{-\phi}\, z_{i,s} \\
m_{i,s} &= \alpha_{i,s}\left(P^{M}_{i,s}\mu\right)^{-\phi} z_{i,s}
\end{aligned}
$$

where $\mu \equiv \frac{\epsilon^{P}}{\epsilon^{P}-1}$.

B.2 Analytical Derivations

We first linearize the CES production function:

$$z_i = \left[\alpha_i^{\frac{1}{\phi}} m_i^{\frac{\phi-1}{\phi}} + (1-\alpha_i)^{\frac{1}{\phi}} \hat{\ell}_i^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}$$

To get:

$$\begin{aligned} dz_i = z_i^{\frac{1}{\phi}} & \left[\omega \left(1-\alpha_i\right)^{\frac{1}{\phi}} \left(\hat{\ell}_i\right)^{\frac{\phi-1}{\phi}-1} \ell_i^{\omega-1} \overline{\ell}_i^{1-\omega} d\ell_i + \alpha_i^{\frac{1}{\phi}} m_i^{\frac{\phi-1}{\phi}-1} dm_i \right] \\ = z_i^{\frac{1}{\phi}} & \left[\omega \left(1-\alpha_i\right) \left(z_i\right)^{\frac{\phi-1}{\phi}} \frac{1}{\left(1-\alpha_i\right) z_i} d\ell_i + \alpha_i \left(z_i\right)^{\frac{\phi-1}{\phi}} \frac{1}{\alpha_i z_i} dm_i \right] \\ dz_i &= d\ell_i + dm_i \end{aligned}$$

The factor demands are:

$$\begin{aligned} \ell_i &= \omega \left(1-\alpha_i\right) \left(\frac{W}{mc_i}\right)^{-\phi} z_i \\ m_i &= \alpha_i \left(\frac{P^M}{mc_i}\right)^{-\phi} z_i \end{aligned}$$

Linearized:

$$\begin{aligned} d\ell_i &= \omega \left(1-\alpha_i\right) dz_i - \phi\omega \left(1-\alpha_i\right) z_i \left(dW - dmc_i\right) \\ dm_i &= \alpha_i dz_i - \phi \alpha_i z_i \left(dP_i^M - dmc_i\right) \end{aligned}$$

Where the price of materials is given by:

$$dP_i^M = \gamma_i dP^{M,F} + \left(1-\gamma_i\right) dP^{M,D}$$

Firms face the following demand curve:

$$z_i = \varrho_i \left(\frac{p_i}{P}\right)^{-\epsilon^P} Z$$

or, in a linearized version:

$$dz_i = -\epsilon^P z_i dp_i + \varrho_i dZ + \epsilon^P z_i dP$$

Finally, firms set prices as a markup over marginal cost:

$$p_i = \mu \times mc_i$$

In the analytical derivations we focus on the limit $\mu \to 1$, though this is not central to our results. This implies $dp_i = dmc_i$.

B.3 Third-Order Approximation of Ξ

We aim for a second-order approximation of Ξ_i which is given by:

$$\Xi_i = \frac{\alpha_i \gamma_i z_i}{\omega \epsilon^P + \phi \left(1 - \omega\right) + \alpha_i \omega \left(\phi - \epsilon^P\right)}$$

For simplicity, we consider an economy with $\gamma_i = \overline{\gamma} \forall i$. We approximate $\Xi_i \left(\alpha_i, z_i\right)$ around a point $(\overline{\alpha}, \overline{z})$. The general expression is:

$$\begin{aligned}
\Xi_i \approx & \overline{\Xi} + \frac{\partial \Xi_i}{\partial z_i} \left(z_i - \overline{z}\right) + \frac{\partial \Xi_i}{\partial \alpha_i} \left(\alpha_i - \overline{\alpha}\right) \\
& + \frac{1}{2!} \frac{\partial^2 \Xi_i}{\partial z_i^2} \left(z_i - \overline{z}\right)^2 + \frac{1}{2!} \frac{\partial^2 \Xi_i}{\partial \alpha_i^2} \left(\alpha_i - \overline{\alpha}\right)^2 + \frac{\partial^2 \Xi_i}{\partial z_i \partial \alpha_i} \left(\alpha_i - \overline{\alpha}\right) \left(z_i - \overline{z}\right) \\
& + \frac{1}{3!} \frac{\partial^3 \Xi_i}{\partial \alpha_i^3} \left(\alpha_i - \overline{\alpha}\right)^3 + \frac{1}{3!} \frac{\partial^3 \Xi_i}{\partial z_i^3} \left(z_i - \overline{z}\right)^3 \\
& + \frac{3}{3!} \frac{\partial^3 \Xi_i}{\partial z_i^2 \partial \alpha_i} \left(\alpha_i - \overline{\alpha}\right) \left(z_i - \overline{z}\right)^2 + + \frac{3}{3!} \frac{\partial^3 \Xi_i}{\partial \alpha_i^2 \partial z_i} \left(\alpha_i - \overline{\alpha}\right)^2 \left(z_i - \overline{z}\right)
\end{aligned}$$

The derivatives are:

$$\frac{\partial \Xi_i}{\partial z_i} = \frac{\alpha_i \overline{\gamma}}{\omega \epsilon^P + \phi\left(1-\omega\right) + \alpha_i \omega \left(\phi - \epsilon^P\right)}$$

$$\frac{\partial \Xi_i}{\partial \alpha_i} = \frac{\overline{\gamma z}\left(\omega \epsilon^P + \phi\left(1-\omega\right)\right)}{\left(\omega \epsilon^P + \phi\left(1-\omega\right) + \alpha_i \omega \left(\phi - \epsilon^P\right)\right)^2}$$

$$\frac{\partial^2 \Xi_i}{\partial z_i^2} = 0$$

$$\frac{\partial^2 \Xi_i}{\partial \alpha_i^2} = 2\frac{\overline{\gamma} z_i \left(\omega \epsilon^P + \phi\left(1-\omega\right)\right)\omega\left(\epsilon^P - \phi\right)}{\left(\omega \epsilon^P + \phi\left(1-\omega\right) + \alpha_i \omega \left(\phi - \epsilon^P\right)\right)^3}$$

$$\frac{\partial^2 \Xi_i}{\partial z_i \partial \alpha_i} = \frac{\overline{\gamma}\left(\omega \epsilon^P + \phi\left(1-\omega\right)\right)}{\left(\omega \epsilon^P + \phi\left(1-\omega\right) + \alpha_i \omega \left(\phi - \epsilon^P\right)\right)^2}$$

$$\frac{\partial^3 \Xi_i}{\partial z_i^2 \partial \alpha_i} = 0$$

$$\frac{\partial^3 \Xi_i}{\partial z_i^3} = 0$$

$$\frac{\partial^3 \Xi_i}{\partial \alpha_i^2 \partial z_i} = 2\frac{\overline{\gamma}\left(\omega \epsilon^P + \phi\left(1-\omega\right)\right)\omega\left(\epsilon^P - \phi\right)}{\left(\omega \epsilon^P + \phi\left(1-\omega\right) + \alpha_i \omega \left(\phi - \epsilon^P\right)\right)^3}$$

$$\frac{\partial^3 \Xi_i}{\partial \alpha_i^3} = 6\frac{\overline{\gamma} z_i \left(\omega \epsilon^P + \phi\left(1-\omega\right)\right)\left(\omega\left(\epsilon^P - \phi\right)\right)^2}{\left(\omega \epsilon^P + \phi\left(1-\omega\right) + \alpha_i \omega \left(\phi - \epsilon^P\right)\right)^4}$$

Combining the terms and using the notation $\overline{\psi} = \omega \epsilon^P + \phi\left(1-\omega\right) + \overline{\alpha}\omega\left(\phi - \epsilon^P\right)$:

$$\begin{aligned}
\Xi_i \approx & \frac{1}{\overline{\psi}}\overline{\alpha}\overline{\gamma} z_i + \frac{\overline{\gamma} z\left(\omega \epsilon^P + \phi\left(1-\omega\right)\right)}{\overline{\psi}^2}\left(\alpha_i - \overline{\alpha}\right) \\
& + \frac{\overline{\gamma z}\left(\omega \epsilon^P + \phi\left(1-\omega\right)\right)\omega\left(\epsilon^P - \phi\right)}{\overline{\psi}^3}\left(\alpha_i - \overline{\alpha}\right)^2 \\
& + \frac{\overline{\gamma}\left(\omega \epsilon^P + \phi\left(1-\omega\right)\right)}{\overline{\psi}^2}\left(\alpha_i - \overline{\alpha}\right)\left(z_i - \overline{z}\right) \\
& + \frac{\overline{\gamma z}\left(\omega \epsilon^P + \phi\left(1-\omega\right)\right)\left(\omega\left(\epsilon^P - \phi\right)\right)^2}{\overline{\psi}^4}\left(\alpha_i - \overline{\alpha}\right)^3 \\
& + \frac{\overline{\gamma}\left(\omega \epsilon^P + \phi\left(1-\omega\right)\right)\omega\left(\epsilon^P - \phi\right)}{\overline{\psi}^3}\left(\alpha_i - \overline{\alpha}\right)^2\left(z_i - \overline{z}\right)
\end{aligned}$$

Aggregating we get:

$$\begin{aligned}
\int \Xi_i \, \mathrm{d}i \approx & \frac{1}{\overline{\psi}} \overline{\alpha} \overline{\gamma} Z + \frac{\overline{\gamma z} \left(\omega \epsilon^P + \phi \left(1 - \omega \right) \right) \omega \left(\epsilon^P - \phi \right)}{\overline{\psi}^4} \mathbb{V}\mathrm{ar} \left(\alpha_i \right) \\
& + \frac{\overline{\gamma} \left(\omega \epsilon^P + \phi \left(1 - \omega \right) \right)}{\overline{\psi}^2} \mathbb{C}\mathrm{ov} \left(\alpha_i, z_i \right) \\
& + \frac{\overline{\gamma z} \left(\omega \epsilon^P + \phi \left(1 - \omega \right) \right) \left(\omega \left(\epsilon^P - \phi \right) \right)^2}{\overline{\psi}^4} \mathbb{V}\mathrm{ar} \left(\alpha_i \right)^{\frac{3}{2}} \mathrm{Skew} \left(\alpha_i \right) \\
& + \frac{\overline{\gamma} \left(\omega \epsilon^P + \phi \left(1 - \omega \right) \right) \omega \left(\epsilon^P - \phi \right)}{\overline{\psi}^3} \int \left(\alpha_i - \overline{\alpha} \right)^2 \left(z_i - \overline{z} \right) \mathrm{d}i
\end{aligned}$$

Note that in order to rewrite the 4th term as function of skewness we require $\mathbb{V}\mathrm{ar}\left(\alpha_i\right) > 0$.

To properly interpret our results, we establish here that $\overline{\psi} > 0$. This is equivalent with:

$$\begin{gathered}
\overline{\psi} > 0 \\
\Leftrightarrow \omega \epsilon^P + \phi \left(1 - \omega \right) + \overline{\alpha} \omega \phi > \overline{\alpha} \omega \epsilon^P
\end{gathered}$$

We first show that this true for $\omega = 0$. In this case the inequality reduces to $\phi > 0$, which we assume to be true. Next consider $\omega = 1$. In this case we have:

$$\overline{\alpha} \phi > - \left(1 - \overline{\alpha} \right) \epsilon^P$$

which is true for $0 < \overline{\alpha} < 1$ and $\phi > 0, \epsilon^P > 0$. Next we confirm that the function $\overline{\psi}\left(\omega\right) = \omega \epsilon^P + \phi \left(1 - \omega\right) - \overline{\alpha} \omega \left(\epsilon^P - \phi\right)$ is increasing everywhere:

$$\begin{aligned}
\overline{\psi}' \left(\omega \right) &= \epsilon^P - \phi - \overline{\alpha} \left(\epsilon^P - \phi \right) \\
&= \epsilon^P - \phi - \overline{\alpha} \epsilon^P + \overline{\alpha} \phi \\
&= \epsilon^P \left(1 - \overline{\alpha} \right) - \phi \left(1 - \overline{\alpha} \right) > 0
\end{aligned}$$

which is increasing if $\epsilon^P > \phi$, which we assume to be true (as is consistent with standard estimates of the two parameters). Given that $\overline{\psi}\left(0\right) > 0, \overline{\psi}\left(1\right) > 0, \overline{\psi}'\left(\omega\right) > 0$, $\overline{\psi}$ is positive.

B.4 A Second-Order Approximation for the Coefficients in dL

In this section we derive a second-order approximation for the firms specific constant Ω_i in the response of labor:

$$d\ell_i = -\phi\left(1-\omega\right)\left(\epsilon^P-\phi\right)\Omega_i dP^{M,F}$$
$$\Omega_i \equiv \left(1-\alpha_i\right)\frac{\left(1-\omega\right)\left(\epsilon^P-\phi\right)}{\epsilon^P\omega+\phi\left(1-\omega\right)-\alpha_i\omega\left(\epsilon^P-\phi\right)}$$

We note that $\Omega_i = \left(1-\alpha_i\right)\Xi_i$, which allows us to reuse derivations form the former section. In particular, we have the following derivatives:

$$\frac{\partial\Omega_i}{\partial z_i} = \left(1-\alpha_i\right)\frac{\partial\Xi_i}{\partial z_i}$$
$$\frac{\partial\Omega_i}{\partial\alpha_i} = \left(1-\alpha_i\right)\frac{\partial\Xi_i}{\partial\alpha_i} - \Xi_i$$
$$\frac{\partial^2\Omega_i}{\partial z_i^2} = \left(1-\alpha_i\right)\frac{\partial^2\Xi_i}{\partial z_i^2}$$
$$\frac{\partial^2\Omega_i}{\partial\alpha_i^2} = \left(1-\alpha_i\right)\frac{\partial^2\Xi_i}{\partial\alpha_i^2} - 2\frac{\partial\Xi_i}{\partial\alpha_i}$$
$$\frac{\partial^2\Omega_i}{\partial z_i\partial\alpha_i} = \left(1-\alpha_i\right)\frac{\partial^2\Xi_i}{\partial z_i\partial\alpha_i} - \frac{\partial\Xi_i}{\partial z_i}$$

Inserting we then have:

$$\begin{aligned}
\Omega_i \approx\ & \overline{\Omega} + \left(1-\alpha_i\right)\frac{\partial\Xi_i}{\partial z_i}\left(z_i-\overline{z}\right) + \left[\left(1-\alpha_i\right)\frac{\partial\Xi_i}{\partial\alpha_i} - \Xi_i\right]\left(\alpha_i-\overline{\alpha}\right) \\
& + \left(1-\alpha_i\right)\frac{\partial^2\Xi_i}{\partial z_i^2}\left(z_i-\overline{z}\right)^2 + \frac{1}{2}\left[\left(1-\alpha_i\right)\frac{\partial^2\Xi_i}{\partial\alpha_i^2} - 2\frac{\partial\Xi_i}{\partial\alpha_i}\right]\left(\alpha_i-\overline{\alpha}\right)^2 \\
& + \left[\left(1-\alpha_i\right)\frac{\partial^2\Xi_i}{\partial z_i\partial\alpha_i} - \frac{\partial\Xi_i}{\partial z_i}\right]\left(\alpha_i-\overline{\alpha}\right)\left(z_i-\overline{z}\right) \\
=\ & \frac{\left(1-\overline{\alpha}\right)\overline{\alpha\gamma}}{\overline{\psi}}z_i + \overline{\gamma z}\left[\frac{\left(1-\overline{\alpha}\right)\left(\omega\epsilon^P+\phi\left(1-\omega\right)\right)-\overline{\alpha}\overline{\psi}}{\overline{\psi}^2}\right]\left(\alpha_i-\overline{\alpha}\right) \\
& + \frac{1}{2}\left[\left(1-\overline{\alpha}\right)2\frac{\overline{\gamma z}\left(\omega\epsilon^P+\phi\left(1-\omega\right)\right)\omega\left(\epsilon^P-\phi\right)}{\overline{\psi}^3} - 2\frac{\overline{\gamma z}\left(\omega\epsilon^P+\phi\left(1-\omega\right)\right)}{\overline{\psi}^2}\right]\left(\alpha_i-\overline{\alpha}\right)^2 \\
& + \left[\left(1-\overline{\alpha}\right)\frac{\overline{\gamma}\left(\omega\epsilon^P+\phi\left(1-\omega\right)\right)}{\overline{\psi}^2} - \frac{\overline{\alpha\gamma}}{\overline{\psi}}\right]\left(\alpha_i-\overline{\alpha}\right)\left(z_i-\overline{z}\right)
\end{aligned}$$

Aggregating:

$$\int \Omega_i di \approx \frac{(1-\overline{\alpha})\,\overline{\alpha\gamma z}}{\overline{\psi}} + \overline{\gamma z}\left[\frac{(1-\overline{\alpha})\left(\omega\epsilon^P + \phi(1-\omega)\right) - \overline{\alpha}\overline{\psi}}{\overline{\psi}^2}\right](\mathbb{E}\,\alpha_i - \overline{\alpha})$$
$$+ \frac{\overline{\gamma z}}{\overline{\psi}^2}\left(\omega\epsilon^P + \phi(1-\omega)\right)\left[\frac{(1-\overline{\alpha})\,\omega\left(\epsilon^P - \phi\right)}{\overline{\psi}} - 1\right]\mathbb{V}\text{ar}(\alpha_i)$$
$$+ \left[(1-\overline{\alpha})\,\frac{\overline{\gamma}\left(\omega\epsilon^P + \phi(1-\omega)\right)}{\overline{\psi}^2} - \frac{\overline{\alpha\gamma}}{\overline{\psi}}\right]\mathbb{C}\text{ov}(\alpha_i, z_i)$$

C Computational Appendix

C.1 Calibration

Name	Definition in data	Descr.	Note.
	Variables		
$Cost_{i,s}$	$= W\ell + P^M M$	Total costs	
$LS_{i,s}$	$= \frac{W\ell}{W\ell + P^M M}$	Labor exp. share	
$p_{i,s}z_{i,s}$	$= p_{i,s}z_{i,s}$	Nominal output/sales	
$\ln pz_{i,s}$	$= \ln p_{i,s}z_{i,s}$	log nominal sales	
Import share	$= \frac{Imports_{i,s}}{P^M m}$	Imports share of intermediate costs	
Export share	$= \frac{Exports_{i,s}}{P_{i,s}z_{i,s}}$	Exports share of sales	
	Moments		
$\frac{\sum_i W_{i,s}\ell_{i,s}}{\sum_i W_{i,s}\ell_{i,s} + P^M_{i,s}M_{i,s}}$		Aggregate labor share in sector s	National Accounts
$\mathbb{V}\text{ar}(LS_{i,s})$		Var. of labor share across all firms	Micro data
$\frac{\sum_i p_{i,s}z_{i,s}}{\sum_s \sum_i p_{i,s}z_{i,s}}$		Sectoral sales share	National Accounts
$\mathbb{V}\text{ar}(\ln pz_{i,s})$		Var. of log sales within sector s	Micro data
$\frac{\sum_i \text{Imports}_{i,s}}{\sum_s P^M_{i,s}M_{i,s}}$		Aggregate import share in sector s	National Accounts
$\mathbb{V}\text{ar}\left(\text{Import share}_{i,s} \mid \text{Imports} > 0\right)$		Var. of import shares amongst importers	Micro data
$\frac{\sum_i \text{Exports}_{i,s}}{\sum_s p_{i,s}z_{i,s}}$		Aggregate export share in sector s	National Accounts
$\mathbb{V}\text{ar}\left(\text{Export share}_{i,s} \mid \text{Exports} > 0\right)$		Var. of export shares amongst exporters	Micro data
$\mathbb{E}\left(\mathbb{1}\left(\left\{\text{Import share}_{i,s} = 0\right\}\right)\right)$		Share with zero imports	Micro data
$\mathbb{E}\left(\mathbb{1}\left(\left\{\text{Export share}_{i,s} = 0\right\}\right)\right)$		Share with zero exports	Micro data
$\text{Corr}(LS_{i,s}, \ln pz_{i,s})$		Corr. between labor share and sales	Micro data
$\text{Corr}\left(\text{Import share}_{i,s}, \ln pz_{i,s} \mid \text{Imports} > 0\right)$		Cond. corr. between import shares and sales	Micro data
$\text{Corr}\left(\text{Export share}_{i,s}, \ln pz_{i,s} \mid \text{Exports} > 0\right)$		Cond. corr. between export shares and sales	Micro data
	Other Moments		
$\text{Corr}\left(\text{Import share}_{i,s}, LS_{i,s} \mid \text{Imports} > 0\right)$		Cond. corr. between import shares and labor shares	Micro data
$\text{Corr}\left(\text{Export share}_{i,s}, LS_{i,s} \mid \text{Exports} > 0\right)$		Cond. corr. between export shares and labor shares	Micro data
$\text{Corr}\left(\text{Export share}_{i,s}, \text{Import share}_{i,s}\right)$		Corr. between import shares and export shares	Micro data

Table A.5: Mapping Between Data and Model Parameters

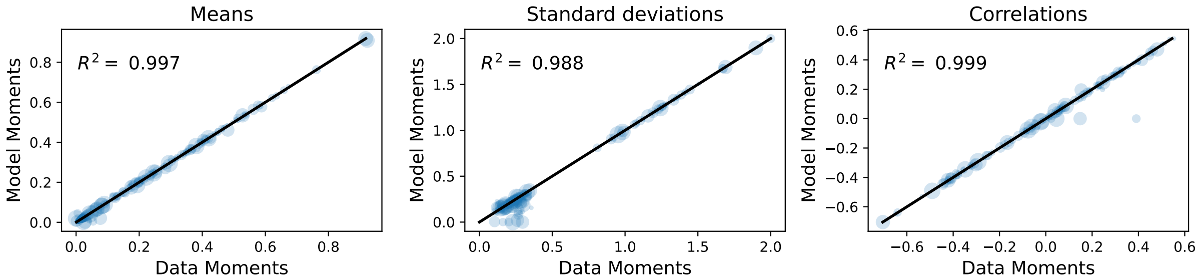

Figure A.11: Model Moments vs. Data Moments in the Calibration

Note: The Figure displays the fitted moments ("internally calibrated" in Table A.5) in the model vs the data. The moments are categorized into means, standard deviations and correlations an pooled across all sectors. The size of dots indicates the size of the associated sector as measured by nominal sales.

D Solution and Calibration Method

D.1 Partial Equilibrium Heterogeneous Firm Impulses

Our heterogeneous firm model contains a large state space because of the 4-dimensional degree of heterogeneity combined with a larger of sectors featured in the model. For this reason, we opt for Monte Carlo methods to solve for impulse responses within each sector. We simulate N firms in each sector using draws from our calibrated distributions, as described in section 4.1.1. With these in hand, we solve - for each firm in N - the firm's problem by linearization. Note that the fact that our model features permanent heterogeneity makes this step trivial, though still time consuming: To solve the firms problem we guess on 4 firm level variables (prices $p_{s,t}$ and factors $\ell_{s,t}, m_{s,t}$) and solve the system of first-order conditions, demand functions and production functions etc. This requires - for each firm $n \in N$ - that we invert a $4 \cdot T \cdot T$ matrix. This inversion is then repeated for each firm and sector, i.e. a total of $N \cdot S$ times. Given the linearized policy functions we then aggregate across firms within each sector - using the relevant CES aggregators when appropriate - to obtain aggregate sectoral responses. That is, we end up with a set of $T \cdot T$ Jacobians for each sector:

$$\left\{ \mathcal{J}_s^{\mathcal{O}_s, \mathcal{I}_s} \right\}_{s \in S}$$

where the sets of inputs and outputs are given by:

$$\mathcal{I}_s = \left\{ W_{s,t}, P_{s,t}^{M,D}, P_{s,t}^{M,F}, P_{s,t}, Z_{s,t}^X, Z_{s,t}^D \right\}_{t=0}^T$$
$$\mathcal{O}_s = \{ P_{s,t}, Z_{s,t}, L_{s,t}, M_{s,t} \}_{t=0}^T$$

D.2 Calibration Details

We calibrate each sector separately to match the various means, variances and correlations described in the main section. We utilize a global optimizer (differential evolution) for this purpose. To polish the global solution we use a root-finder, or a Nelder-Mead algorithm. We have experimented with various objective function for the minimization problem, but have found that an R^2-measure works well. That is, we solve:

$$\min_{\{x\}} \left| R^2\left(y, y^{Data}\right) - 1 \right|$$

where:

$$R^2\left(y, y^{Data}\right) 1 - \left(\frac{\sum_i \omega_i \cdot \left(y_i - y_i^{Data}\right)^2}{\sum_i \omega_i \cdot \left(y_i - \sum_i y_i \omega_i\right)^2} \right)$$

and where y_i is the $i'th$ moment in the model, y_i^{Data} is the i'th moment in the data, and ω_i denotes subject weights. To improve convergence, and allow for the use of derivative-based optimizers we ensure that the objective function is smooth.

D.3 Impulse-Response Function Matching

Empirical impulse-responses To estimate the causal effect of foreign supply shocks, we apply the estimation framework outlined in Section 3. We estimate an equation identical to (11), except we leave out the interaction term implying that we only identify the average effect, corresponding to the model counterpart we try to match. As before, we include sector interacted with time fixed effects implying that we identify a partial equilibrium response.

Matching to empirical moments We apply the following parameters to match the empirical responses to the foreign supply shock: The adjustment costs on labor and materials, the elasticity of substitution between the two, and the elasticity of substitution between varieties within sectors. Collecting the parameters in a vector $\Psi = (\phi^L, \phi^M, \phi, \epsilon^P)$, and letting $\hat{\boldsymbol{J}}$ denote the set of empirical impulse responses we want to match and $\boldsymbol{J}(\Psi)$ the model counterpart which depends on the deep parameters in Ψ, the estimated parameter values solve the minimization problem:

$$\min_{\Psi} \left(\boldsymbol{J}(\Psi) - \hat{\boldsymbol{J}}\right)' \Sigma^{-1} \left(\boldsymbol{J}(\Psi) - \hat{\boldsymbol{J}}\right)$$

where Σ is a diagonal matrix containing the variances of the estimated impulse responses in $\hat{\boldsymbol{J}}$. For the procedure, we include in $\hat{\boldsymbol{J}}$ the responses of firm prices, real output, value-added, labor expenditures, and materials expenditure.

D.4 Solving for General Equilibrium

We solve for general equilibrium as in Auclert et al. (2021). We write the model in sequence space and represent the model as a residual $\boldsymbol{H}$ which depends on unknowns $\boldsymbol{X}$ and shocks $\boldsymbol{Z}$. The model solution is characterized by:

$$\boldsymbol{H}(\boldsymbol{X}, \boldsymbol{Z}) = \mathbf{0} \tag{42}$$

Example: In our multi-sector model the residual $\boldsymbol{H}$ contains the goods market residuals for the domestic and exporting markets, the UIP condition and the sectoral NKWPC, while the unknowns are sectoral price for each goods market, the nominal exchange rate and nominal sectoral wages. Hence $\boldsymbol{H}$ and $\boldsymbol{X}$ are of dimension $3T \cdot S + T$. The shocks $\boldsymbol{Z}$ is a sequence $3 \cdot T$ of shocks to exogenous import prices, foreign demand and foreign interest rates. Eq. (42) then amounts to solving $3T \cdot S + T$ equations in $3T \cdot S + T$ unknowns.

Linearizing (42) and solving for the unknowns $\boldsymbol{X}$ we obtain the linearized solution:

$$d\boldsymbol{X} = -\boldsymbol{H}_X^{-1}\boldsymbol{H}_Z d\boldsymbol{Z}$$

where $\boldsymbol{H}_Z, \boldsymbol{H}_Z$ are the general equilibrium Jacobians of $\boldsymbol{H}$ w.r.t $\boldsymbol{X}$ and $\boldsymbol{Z}$ respectively. We compute these using standard numerical methods.

In practice we obtain $\boldsymbol{H}_X, \boldsymbol{H}_Z$ by splitting the model into a pre-firm block and a post-firm block. Given unknowns $\boldsymbol{X}$ and shocks $\boldsymbol{Z}$ we can evaluate the pre-firm block. Given $\boldsymbol{X}, \boldsymbol{X}$ and the output of pre-firm block we can evaluate the sectoral responses using the sectoral Jacobians $\left\{\mathcal{J}_s^{\mathcal{O}_s,\mathcal{I}_s}\right\}_{s\in S}$. Given the sectoral responses we evaluate the-post firm block to obtain the targets $\boldsymbol{H}$. We use this procedure to evaluate the model. We obtain $\boldsymbol{H}_X, \boldsymbol{H}_Z$ by shock each of the inputs in $\boldsymbol{X}, \boldsymbol{H}$ at each time t separately and using the standard numerical approximation for the Jacobian:

$$\boldsymbol{H}_X = \frac{\boldsymbol{H}(\boldsymbol{X} + h) - \boldsymbol{H}(\boldsymbol{X})}{h}, \boldsymbol{H}_Z = \frac{\boldsymbol{H}(\boldsymbol{Z} + h) - \boldsymbol{H}(\boldsymbol{Z})}{h}$$

with $h = 1\text{e}{-04}$.

E GE Responses to SVAR Supply Shock

In this section, we consider the response of the Danish economy to the full estimated foreign supply shock. Figure A.12 displays the response of domestic key variables to the aggregate foreign supply shock. The foreign shock generates what resembles a supply shock in the domestic economy: There is a persistent decline in GDP and employment, with a simultaneous increase in domestic CPI lasting for about 2 years. The real interest rate increases initially to bring down domestic inflation, which causes an initial decline in domestic consumption in the baseline model due to intertemporal substitution. The presence of nominal wage rigidities implies that real wages decline in the face of higher prices. This causes firms to substitute toward labor, thus alleviating some of the decline caused by lower demand.

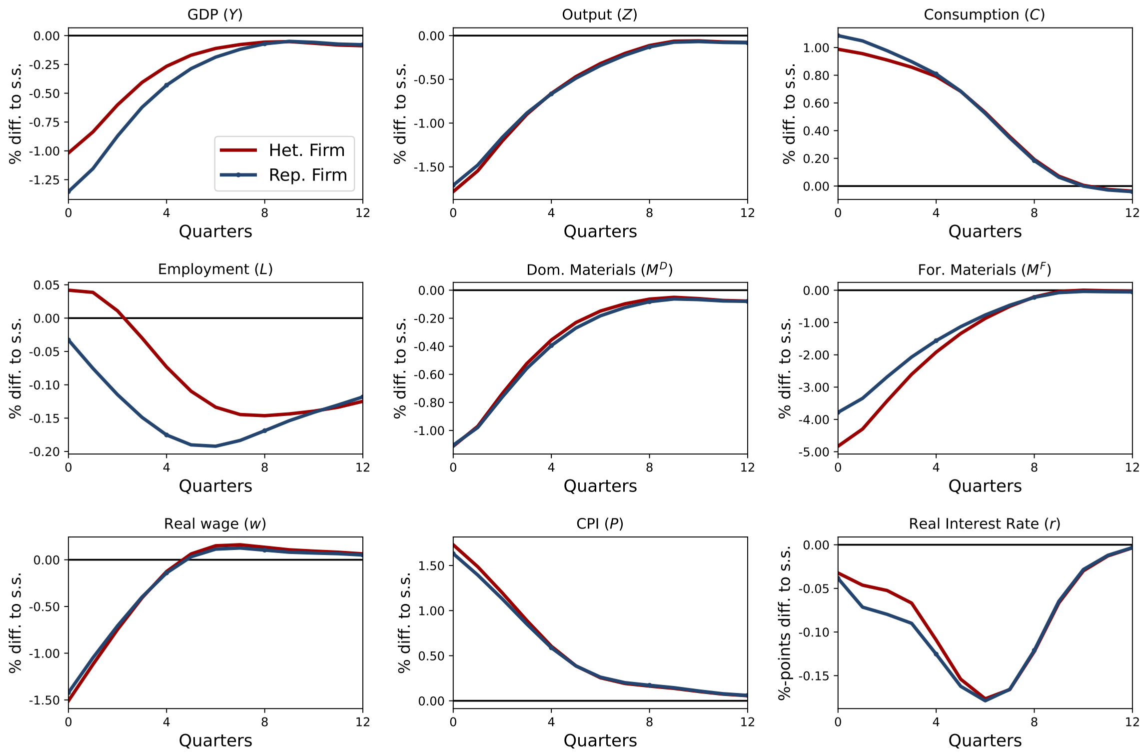

Figure A.12: Domestic Responses to Aggregate Foreign Supply Shock (SVAR)

Note: Domestic responses to foreign supply shocked based on the estimated SVAR model. The shock is scaled such that foreign demand drops by 1% on impact.

To more clearly understand what drives our aggregate responses, we consider two decompositions. The first decomposition decomposes our aggregate supply shock into contributions coming from foreign prices, the foreign interest rate and foreign demand. In Figure A.13, we present this decomposition for GDP, output, and producer price inflation. The decomposition highlights that the main driver of the decline in domestic GDP and output and the increase in producer price inflation comes from the higher import prices faced by firms and households. There is a small but significant contribution from foreign demand, while the shock to the foreign interest rate stabilizes domestic GDP and inflation. This derives from the expenditure channel of exchange rates. When the foreign interest rate decreases, capital flows imply an appreciation of the domestic exchange rate vis-à-vis foreign countries, which makes imports cheaper thereby stabilizing the domestic economy.

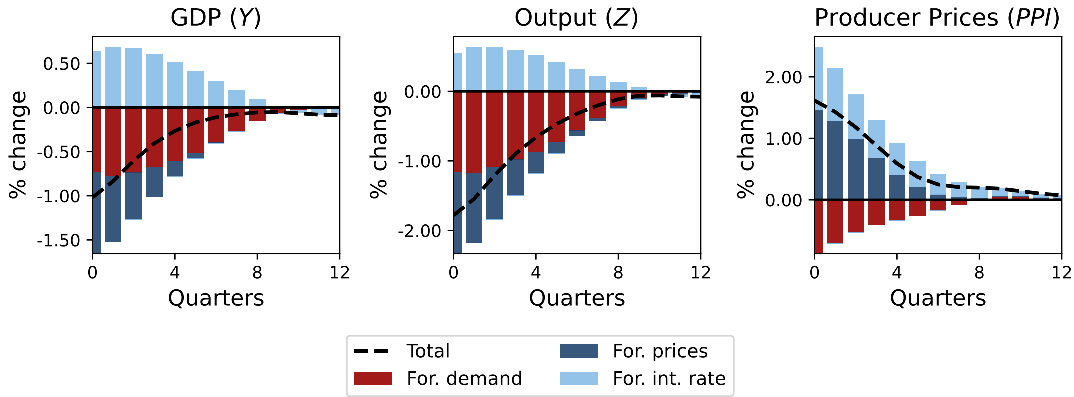

Figure A.13: Aggregate Output and Prices Decomposed in Aggregate SVAR Shocks

Note: This Figure decomposes the response of aggregate GDP and inflation into contributions from the various aggregate shocks in Figure A.3. Since we linearize the model w.r.t aggregate shocks, the total effect is the sum of the individual shocks.

The next decomposition we consider relates to what drives the supply side of our model. Figure A.14 decomposes the aggregate response of GDP, output, and producer price inflation into contributions from the various factors that influence firm behavior in our model. The decline in output is driven by lower demand as well as higher import prices. This decline in output is mitigated by lower domestic prices of materials as well as declining real wages. We find only a minor effect on output of changes in competing prices within sectors, but they do reflect a substantial part of the increase in the producer price index. The drop in GDP is, in particular, driven by decreasing competitors' prices, which makes the individual firm less competitive.

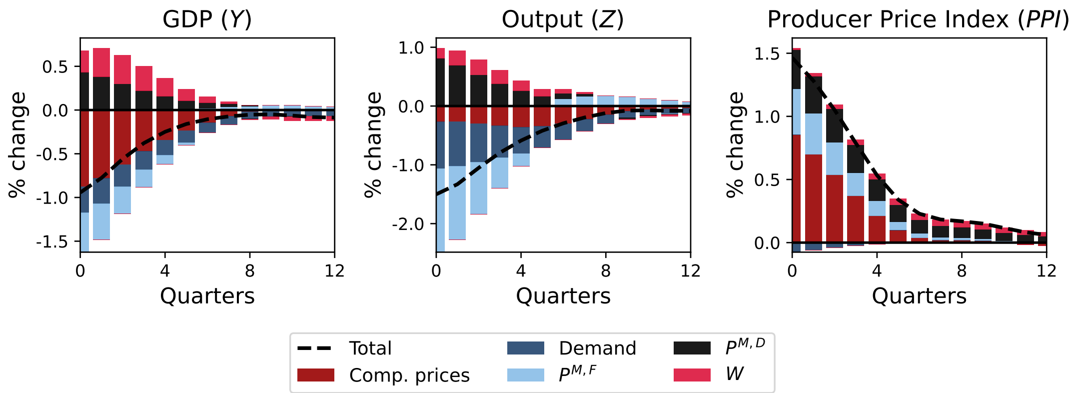

Figure A.14: Aggregate Output and Prices Decomposed by Firm-Level Inputs (SVAR Shock)

Note: This Figure decomposes the response of output and inflation in the general equilibrium model into contributions from the various firm inputs. "Demand" refer to effects from $Z^D_{s,t}$, $Z^X_{s,t}$, "competitors price" refers effects from sectoral prices $P_{s,t}$.

E.1 The Role of Heterogeneity

In Figure A.12, we display the responses in the model with within-sector heterogeneity compared to the model with only sectoral heterogeneity. The model with only sectoral heterogeneity predicts a *larger* and persistent drop in GDP and employment. As the inflation response in the model with only sectoral heterogeneity is lower than the baseline model, it implies that the real interest rate increases less, thereby increasing consumption compared to the baseline model. The model with within-sector heterogeneity implies a larger drop in expenditure on foreign materials, reflecting that larger firms are relatively more connected to the foreign economy through production networks and more likely to pass on the shock upstream to the foreign economy.

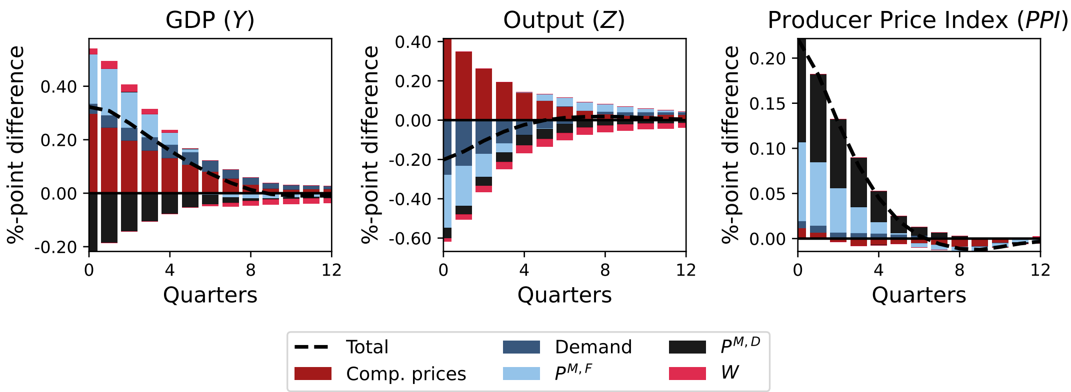

Figure A.15: Decomposition of Model Differences for Aggregate GDP, Output and Inflation (SVAR Shock)

Note: This Figure shows decomposed differences between our two models. We plot the differences in GDP, output and PPI inflation coming from contributions from the various firm inputs. "Demand" refer to effects from $Z^D_{s,t}$, $Z^X_{s,t}$, "competitors price" refers effects from sectoral prices $P_{s,t}$.

To elaborate further on the exact mechanisms behind the dampening in GDP but amplification in output and inflation of within-sector heterogeneity, in Figure A.15, we decompose the %-point difference between the two models into the contributions from the firm input. As previously, the amplification of output is driven by the increase in foreign import prices and demand. The amplification is partly offset by an increase in the competitor's prices. The dampening in GDP is mainly driven by the increase in import prices, which mainly affects the larger firms in the baseline model and creates a dampening in GDP because the labor-intensive firms are less affected by the foreign shock. In addition, as for output, higher competitors prices dampens the impact of the foreign shock as it allows other firms to increase its sales without experiencing a drop in demand. The dampening is partly offset by the increase in material prices in the baseline model exceeding the response in the model with no sectoral heterogeneity, which is a consequence of the heterogeneity in material shares.

F General Equilibrium Model Robustness

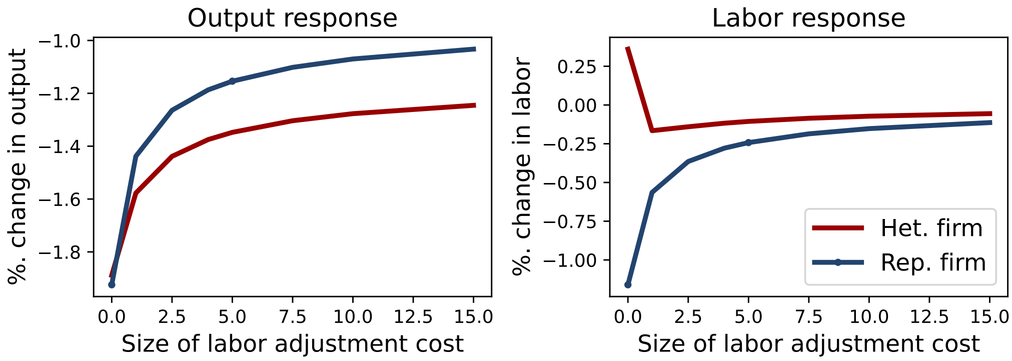

Figure A.16: GE Output and Labor Responses for Varying Labor Adjustment Cost

Note: The figure displays the output and labor responses in GE in the first period after the stylized supply shock in the heterogeneous and representative firm model.

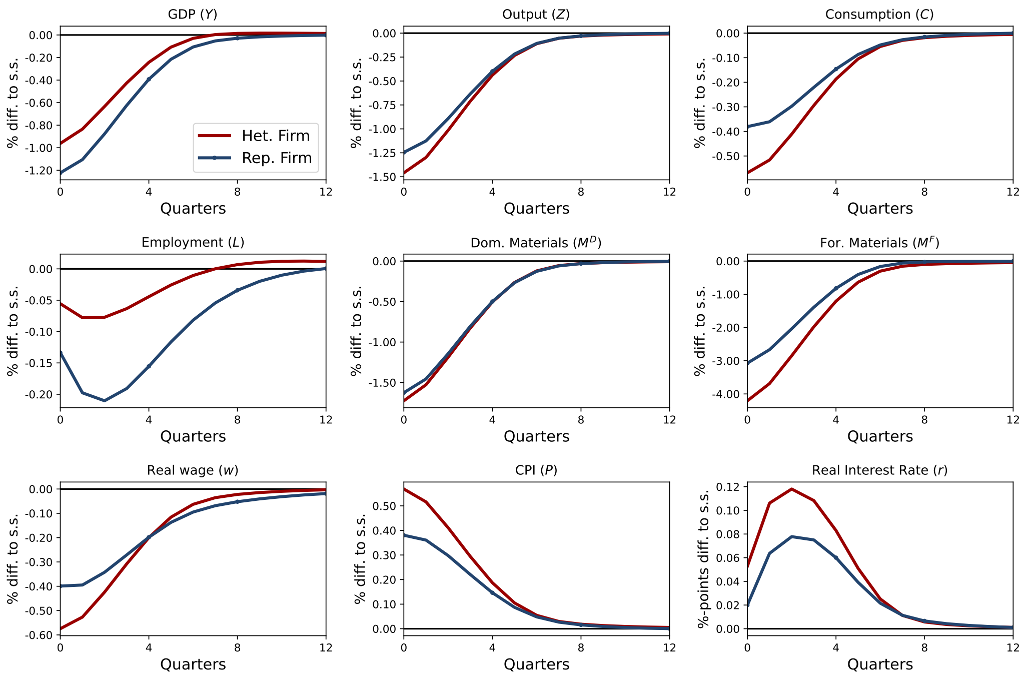

Figure A.17: Responses to an Aggregate Foreign Supply Shock with Substitutable Intermediate Goods

Note: General equilibrium impulse responses to a 3% increase in import prices with $\eta = 2$ up from $\eta = 0.2$, implying that material inputs from different sectors are more substitutable.

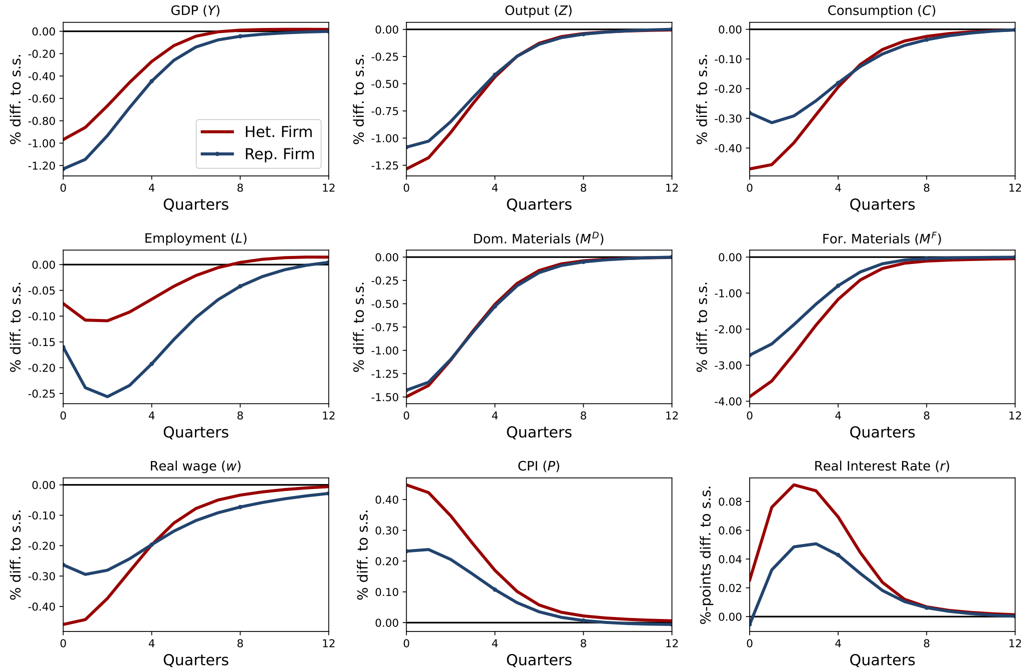

Figure A.18: Responses to an Aggregate Foreign Supply Shock with Hand-to-Mouth Households

Note: General equilibrium impulse responses to a 3% increase in import prices with a share $\lambda = 0.5$ of hand-to-mouth households in the model, implying an aggregate MPC out of real labor income of 50%.

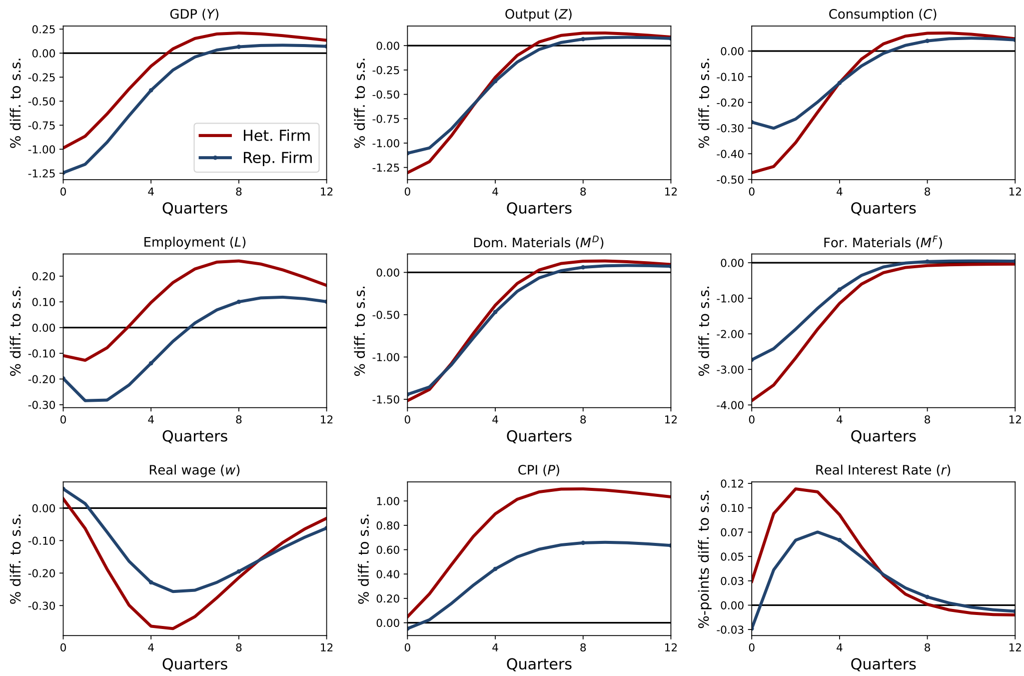

Figure A.19: Responses to Aggregate Foreign Supply Shock with a Floating Exchange Rate

Note: General equilibrium impulse responses to a 3% increase in import prices with a floating exchange rate. Domestic monetary policy follows a standard Taylor rule.

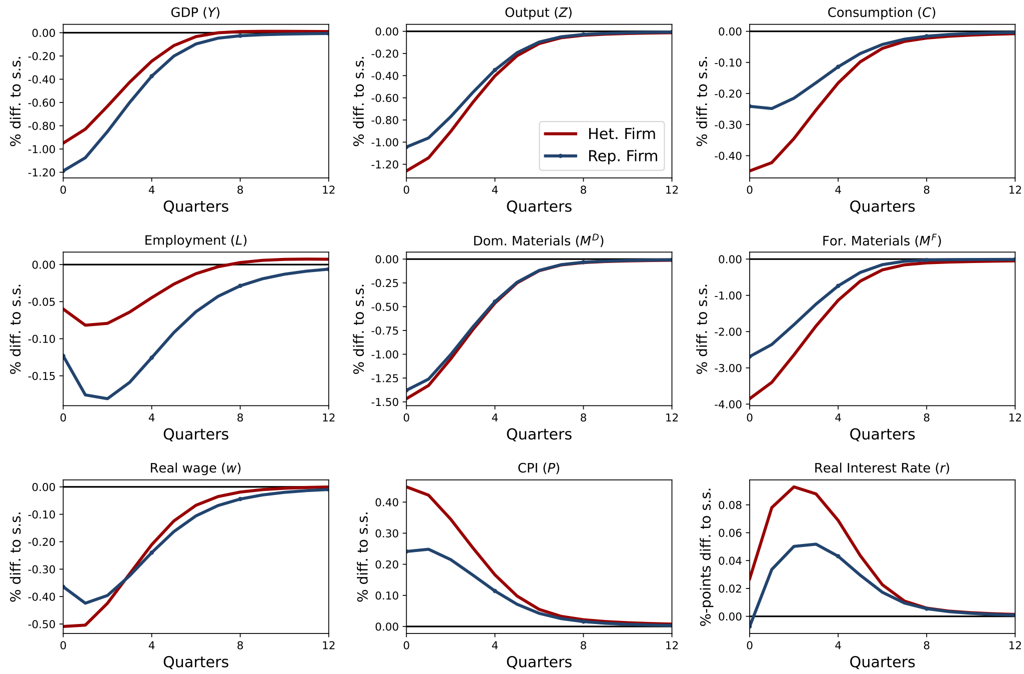

Figure A.20: Responses to Aggregate Foreign Supply Shock with Flexible Wages

Note: General equilibrium impulse responses to a 3% increase in import prices with completely flexible wages, $\kappa^W \to \infty$.